Imaginary / Complex Numbers

* Recall, if we square any number, the result is always positive,

Let $a =$ positive integer

Then $a^2 = a^2$
$(-a)^2 = a^2$] outcome is always positive

→ Arguably, no negative number can have a square root.

The imaginary number $i = \sqrt{-1}$

$\sqrt{-b}$: if b is a positive number then $\sqrt{-b} = \sqrt{b}\, i$

eg. $\sqrt{-64} = \sqrt{64}\, i$
$= 8i$

$-\sqrt{-4} = -1\sqrt{4}\, i$
$= -2i$

$\sqrt{-50} = \sqrt{50}\, i$
$= \sqrt{25}\sqrt{2}\, i$
$= 5\sqrt{2}\, i$

$\sqrt{-29} = \sqrt{29}\, i$

Simplifying quotients in terms of i

$$\frac{\sqrt{-100}}{\sqrt{-25}} = \frac{\sqrt{100}\, i}{\sqrt{25}\, i} = \frac{10\cancel{i}}{5\cancel{i}} = 2$$

* Note: if radicands are negative, then you must work with radicals first

$$\frac{\sqrt{-100}}{\sqrt{-25}} \neq \sqrt{\frac{-100}{-25}}$$ * only if radicands are positive.

$$\begin{aligned} \sqrt{-25}\,\sqrt{-9} &= \sqrt{25}\, i \cdot \sqrt{9}\, i \\ &= (5i)(3i) \\ &= 15i^2 \\ &= 15(-1) \\ &= -15 \end{aligned}$$

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$i^2 = -1$

Powers of i

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -1 \cdot i = -i$

$i^4 = -i\, i = 1$

$= -i^2$

$= -(-1)$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

$i^{11} = -i$

$i^{12} = 1$

Patterns: cycles every 4th power.

$i^{4n} = 1 \quad , n \in \{0, \pm 1, \pm 2, \ldots\}$

$n \in \mathbb{Z}$

$i^{4n+1} = i^1 = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -i$

$i^{4n+0} = i^0 = i^4 = 1$

$i^{\text{even power}} = \pm 1$

$i^{2(\text{odd \#})} = -1 \qquad i^{2(\text{even \#})} = 1$

$$i^{18} = i^{16+2} = i^2 = -1$$

using cycles of 4 — largest number less than 18 divisible by 4

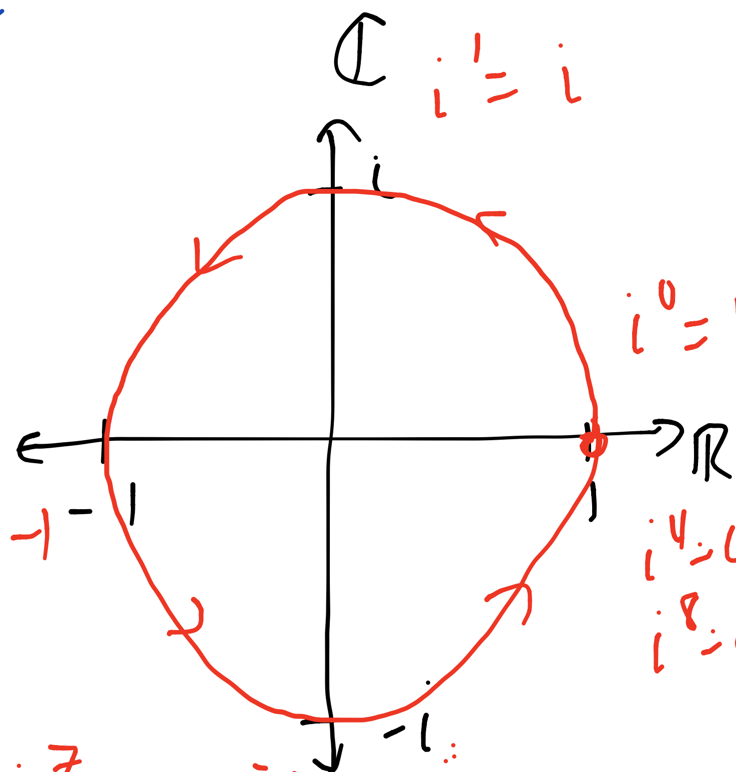
$$i^{18} = (i^2)^9 = (-1)^9 = -1$$

power rule for exponents

$$i^{23} = i^{22}\, i^1 = (i^2)^{11}\, i$$
$$= (-1)^{11}\, i$$
$$= -1\, i$$
$$= -i$$

$i^5 = i$

$i^1 = i$



$i^0 = 1$

$i^4 = 0$

$i^8 = 0$

$i^6 = i^2 = -1$

$i^7 = i^3 = -i$

Complex Numbers

are of the form $a+bi$

a is real component

bi is imaginary component.

$a, b \in \mathbb{R}$ "real numbers"

$i = \sqrt{-1}$

* if $b=0$, $a+bi$ is real number
  $b \neq 0$, $a+bi$ is imaginary number

* Recall. $a^2-b^2 = (a-b)(a+b)$

  → difference of squares

  → $a-b$ and $a+b$ are "conjugates"

  → the factors of a difference of squares are conjugates of each other.

Recall

a^2+b^2 is not factorable at this point

- b/c not like terms
- not difference of squares

Let's multiply two complex conjugates

$$(a+bi)(a-bi) = a^2-(bi)^2$$
$$= a^2-b^2i^2$$
$$= a^2-b^2(-1)$$
$$(a+bi)(a-bi) = a^2+b^2$$

e.g.

$$x^2+36 = x^2+(6)^2$$
$$= (x+6i)(x-6i)$$