

Imaginary / Complex Numbers

* Recall if we square any number, the result is always positive,

Let $a = \text{positive integer}$

$$\begin{aligned} \text{Then } a^2 &= a^2 \\ (-a)^2 &= a^2 \end{aligned} \quad \left. \begin{array}{l} \text{outcome is always} \\ \text{positive} \end{array} \right\}$$

→ Arguably, no negative number can have a square root.

The imaginary number $i = \sqrt{-1}$

$\sqrt{-b}$. if b is a positive number

$$\text{then } \sqrt{-b} = \sqrt{b} i$$

$$\begin{array}{c|c} \text{e.g. } \sqrt{-64} = \sqrt{64} i & -\sqrt{-4} = -1 \sqrt{4} i \\ = 8i & = -2i \end{array}$$

$$\begin{array}{c|c} \sqrt{-50} = \sqrt{50} i & \sqrt{-29} = \sqrt{29} i \\ = \sqrt{25} \sqrt{2} i & \\ = 5\sqrt{2} i & \end{array}$$

Simplifying quotients in terms of i

$$\frac{\sqrt{-100}}{\sqrt{-25}} = \frac{\sqrt{100}i}{\sqrt{25}i} = \frac{10i}{5i} = 2$$

* Note: if radicands are negative, then you must work with radicals first

$$\frac{\sqrt{-100}}{\sqrt{-25}} \neq \sqrt{\frac{-100}{-25}} \quad * \text{only if radicands are positive.}$$

$$\begin{aligned}\sqrt{-25} \sqrt{-9} &= \sqrt{25}i \cdot \sqrt{9}i \\&= (5i)(3i) \\&= 15i^2 \\&= 15(-1) \\&= -15\end{aligned}\quad \left| \begin{array}{l} i = \sqrt{-1} \\ i^2 = (\sqrt{-1})^2 \\ i^2 = -1 \end{array} \right.$$

Powers of i

$$\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -1 \cdot i = -i \\ i^4 &= -i \cdot i = 1 \\ &= -i^2 \\ &= -(-1) \end{aligned}$$

$$\begin{array}{lll} i^5 = i & i^9 = i \\ i^6 = -1 & i^{10} = -1 \\ i^7 = -i & i^{11} = -i \\ i^8 = 1 & i^{12} = 1 \end{array}$$

Patterns: cycles every 4th power.

$$i^{4n} = 1, \quad n \in \{0, \pm 1, \pm 2, \dots\}$$

$$i^{4n+1} = i$$

$$i^{4n+2} = i^2 = -1$$

$$i^{4n+3} = i^3 = -i$$

$$i^{4n+0} = i^0 = i^4 = 1$$

$$i^{\text{even power}} = \pm 1$$

$$i^{2(\text{odd } \#)} = -1 \quad i^{2(\text{even } \#)} = 1$$

$$i^{18} = i^{16+2} = i^2 = -1$$

using cycles of 4 - largest number less than 18

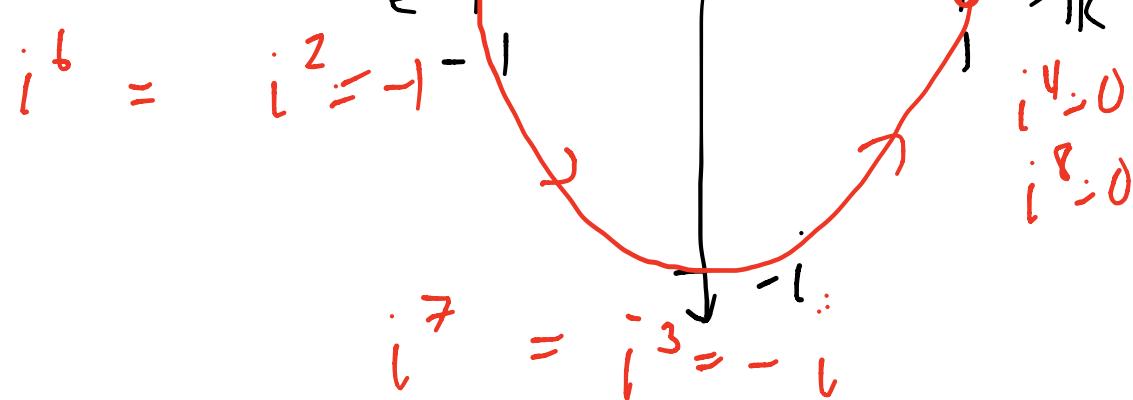
$$i^{18} = (i^2)^9 = (-1)^9 = -1 \quad \text{divisible by 4}$$

power rule for exponents

$$\begin{aligned} i^{23} &= i^{22} \cdot i^1 = (i^2)^{11} \cdot i \\ &= (-1)^{11} \cdot i \\ &= -1 \cdot i \\ &= -i \end{aligned}$$

$$i^5 = i$$

$$\textcircled{1} \quad i^1 = i$$



Complex Numbers are of the form $a+bi$

a is real component

bi is imaginary component.

$a, b \in \mathbb{R}$ "real numbers")

$$i = \sqrt{-1}$$

* if $b=0$, $a+bi$ is real number

$b \neq 0$, $a+bi$ is imaginary number

* Recall. $a^2 - b^2 = (a-b)(a+b)$

→ difference of squares

→ $a-b$ and $a+b$ are "conjugates"

→ the factors of a difference of squares
are conjugates of each other.

Recall

$a^2 + b^2$ is not factorable at this point

- b/c not like terms

- not difference of squares

Let's multiply two complex conjugates.

$$(a+bi)(a-bi) = a^2 - (bi)^2$$

$$= a^2 - b^2 i^2$$

$$= a^2 - b^2(-1)$$

$$(a+bi)(a-bi) = a^2 + b^2$$

e.g.

$$\begin{aligned}x^2 + 36 &= x^2 + (6)^2 \\&= (x+6i)(x-6i)\end{aligned}$$

Adding and Subtracting Complex Numbers

$$(1 - 5i) + (-3 + 7i)$$

$$1 - 5i \quad -3 + 7i$$

(broke the parentheses)

$$1 - 3 - 5i + 7i$$

grouped constants

$$-2 + 2i$$

grouped imaginary #'s
added like terms

$$(-6 - 11i) - (-9 - 12i)$$

$$(-6 - (-9)) \quad ((-11i) - (-12i))$$

$$(-6 + 9) \quad (-11i + 12i)$$

$$3 \quad + \quad 1i \quad ?$$

$$(-6 - 11i) - (-9 - 12i)$$

Preferred
to KCF

$$(-6 - 11i) + (9 + 12i)$$

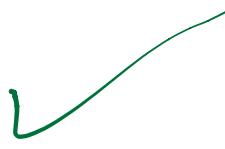
"Keep Change Flip"

$$-6 - 11i + 9 + 12i$$

$$-6 + 9 - 11i + 12i$$

$$3 + 1i$$

$$3 + i$$



$$7b. (5+9i) - (-2+3i)$$

$$5+9i + 2-3i$$

$$5+2+9i-3i$$

$$7+6i$$

$$(-5+9i) - (-2+3i)$$

$$\cancel{-5+9i} \quad \cancel{+2-3i}$$

$$\cancel{-3} \quad +6i$$

7c.

$$4i \left(6 - \frac{11}{16}i \right)$$

$$(4i)(6) - (4i)\left(\frac{11}{16}i\right)$$

$$\ast \quad 1 \cancel{4} \cdot \cancel{\frac{11}{16}}_4 = \frac{11}{4}$$

$$4i \cdot 6 =$$
$$4 \cdot 6i = 24i$$

$$24i - \frac{11}{4}i^2$$

$$i^2 = -1$$

$$24i - \frac{11}{4}(-1)$$

$$+ 24i + \frac{11}{4}$$

$$\frac{11}{4} + 24i$$

← Need to write complex #'s
in $a+bi$ form

* by commutative property
of addition

$$a+b = b+a$$

$$(10 - 5i)(2 + 3i)$$

$$(10)(2) + (-5i)(2) + (3i)(10) + (-5i)(3i)$$

$$20 - 10i + 30i - 15i^2$$

$$20 - 10i + 30i + 15$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 20 + 15 & -10i + 30i \end{array}$$

$$\begin{aligned} i^2 &= -1 \\ -15(-1) &= +15 \end{aligned}$$

$$35 + 20i$$

$a+bi$ form

$$(3+4i)(3-4i)$$

$$\text{"a}^2 - b^2\text{"}$$

$$9 - 16i^2$$

$$9 - 16(-1)$$

$$9 + 16 = 25$$

$$(3)(3) + (3)(-4i) + (4i)(3) + (4i)(-4i)$$

$$9 - 12i + 12i - 16i^2$$

$$9 - 16i^2$$

$$9 - 16(-1)$$

$$9 + 16 = 25$$

Dividing Complex Numbers

$$i^2 = -1$$

$$i = \sqrt{-1}$$

$$\frac{4-3i}{5+2i}$$

$$= \frac{4-3\sqrt{-1}}{5+2\sqrt{-1}} = \frac{4-3\sqrt{-1}}{5+2\sqrt{-1}} \left(\frac{5-2\sqrt{-1}}{5-2\sqrt{-1}} \right)$$

multiplied by the conjugate
of $5+2\sqrt{-1}$.. $5-2\sqrt{-1}$

$$5+2i \quad 5-2i$$

$$\left(\frac{4-3i}{5+2i} \right) \left(\frac{5-2i}{5-2i} \right)$$

1. Multiply by conjugate
over itself

2. multiply numerator
and denominator separately
.. "distribute"

$$\frac{(4)(5) + (-3i)(-2i) + (4)(-2i) + (-3i)(5)}{(5)(5) + (5)(2i) + (2i)(5) + (2i)(-2i)}$$

$$20 - 8i - 15i + 6i^2$$

$$25 \cancel{- 10i} \quad \cancel{+ 10i} \quad - 4i^2$$

$$\begin{array}{r} 20 - 6 \\ \hline 25 + 4 \end{array}$$

3. Simplify

$$\frac{14 - 23i}{29}$$

$$\frac{14}{29} - \frac{23i}{29}$$

$$\frac{14}{29} - \frac{23}{29} i$$

4. a+bi form