1. Definition. <u>square root</u> –

a square root of 25 = 5

a square root of 25 = −5

$\sqrt{49}$ = 7, −7

$\sqrt{0}$ = 0 ← 0 is neither positive nor negative

$\sqrt{-9}$ = no real root ← imaginary number

Definition. <u>square root</u> – b is a square root of a if $b^2 = a$.

→ $\sqrt{a}$: the principal/positive square root of a.

→ $-\sqrt{a}$: the negative square root of a.

examples

$\sqrt{36} = 6$

$\sqrt{121} = 11$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

$\frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

$\sqrt{-144} =$ imaginary

$-\sqrt{144} = -12$

$\sqrt{.04} = .2 = \sqrt{\frac{4}{100}} = \sqrt{\frac{1}{25}} = \frac{1}{5} = .2$

$(.2)(.2) = .04$

$-\sqrt{-0.01} =$ imaginary

$-\sqrt{\frac{1}{9}} = -\frac{1}{3}$

2. Definition. n^{th} root.

e.g. 2 is a square root of 4. $2^2 = 4$

2 is a cube root of 8 $2^3 = 8$

" " " "third root" of 8.

2 is a fourth root of 16 $2^4 = 16$

Definition. n^{th} root. b is nth root of a if $b^n = a$

$\sqrt[n]{a}$

n = index → if n is invisible, assume square root.

a = radicand → if not, then assume nth-root.

→ $n \neq 1$

$\sqrt[4]{81} \Rightarrow 3^4 = 81 \rightarrow \sqrt[4]{81} = 3$

$\sqrt[4]{625} \rightarrow 5^4 = 625 \rightarrow \sqrt[4]{625} = 5$

$\sqrt[4]{-625} \rightarrow$ imaginary

$\sqrt[3]{27} \Rightarrow 3^3 = 27 \rightarrow \sqrt[3]{27} = 3$

$\sqrt[3]{125} \rightarrow 5^3 = 125 \rightarrow \sqrt[3]{125} = 5$

$\sqrt[3]{-27} \rightarrow (-3)^3 = -27 \rightarrow \sqrt[3]{-27} = -3$

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odd number of negative signs

$(-3)(-3)(-3) =$

$(+)\cdot(-) = -$

$\sqrt[3]{-125} = -5$

$\sqrt[5]{32} \Rightarrow 2^5 = 32 \rightarrow \sqrt[5]{32} = 2$

$\sqrt[5]{-32} \Rightarrow (-2)^5 = -32 \rightarrow \sqrt[5]{-32} = -2$

$\sqrt[n]{a}$ – cannot find real root if n is even and a is negative
(n: index; a: radicand)

– if n is odd and a is positive
→ $\sqrt[n]{a}$ is positive

– if n is odd and a is negative
→ $\sqrt[n]{a}$ is negative.

– if n is even, then $\sqrt[n]{a}$ is the principal (positive) nth root of a

– if n is odd, then $\sqrt[n]{a}$ is the n-th root of a.

Evaluating $\sqrt[n]{a^n}$ - if n is odd, then

$$\sqrt[n]{a^n} = a$$

- if n is even, then

$$\sqrt[n]{a^n} = |a|$$

$$\sqrt[8]{(78)^8} = |78| = 78$$

$$\sqrt[5]{(3)^5} = 3 \qquad \sqrt[5]{(-3)^5} = -3$$

$$\sqrt[4]{(3)^4} = |3| = 3$$

$$\sqrt[4]{(-3)^4} = |-3| = 3$$

$$\sqrt[2]{(x+2)^2} = |x+2|$$ ← index is even, need absolute value

$$\sqrt[3]{(x+2)^3} = x+2$$ ← index is odd, don't need.

$$\sqrt[4]{x^4} = |x|$$

$$\sqrt[8]{(a+b)^8} = |a+b|$$

$$\sqrt[7]{(a+b)^7} = a+b$$

$\sqrt{x^4} = \sqrt{(x^2)^2} = |x^2| = x^2$ because $x^2 \geq 0$

$\sqrt[3]{x^9} = \sqrt{(x^3)^3} = x^3$

$\sqrt{x^6} = \sqrt{(x^3)^2} = |x^3|$ ← keep absolute value b/c x^3 can be negative

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Radical Functions: $f(x) = \sqrt[n]{x}$ is called a radical function

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Rational Exponents:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{1}{3}} = \sqrt[3]{a}$$

$$\sqrt[5]{a} = a^{\frac{1}{5}}$$

$$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

$$= \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$* \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$$7^{\frac{4}{5}} = \sqrt[5]{7^4} = \left(\sqrt[5]{7}\right)^4$$

$$(2a+3)^{\frac{3}{2}} = \sqrt{(2a+3)^3} = \left(\sqrt{2a+3}\right)^3$$