Multiplying & Dividing Rational Expressions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$ (Multiplying Fractions)

Analyze

$$\frac{6x^3}{5} \cdot \frac{2}{3x} = \frac{12x^3}{15x} = \frac{3x \cdot 4x^2}{3x \cdot 5} = (1)\frac{4x^2}{5} = \frac{4x^2}{5}$$

Recall $ab = ba$ ← commutative property of multiplication

$$\frac{6x^3}{5} \cdot \frac{2}{3x} = \frac{2}{5} \cdot \frac{6x^3}{3x}$$

$$= \frac{2}{5} \cdot \frac{3x \cdot 2x^2}{3x \cdot 1}$$

$$= \frac{4x^2}{5}$$

$$\frac{6x^3}{5} \cdot \frac{2}{3x} = \frac{\cancel{6x^3}^{2x^2}}{5} \cdot \frac{2}{\cancel{3x}_{1}} = \frac{4x^2}{5}$$

* cross canceling - only works when multiplying fractions

Division of fractions *KCF - Keep change Flip

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{c}{d}$$

$$\frac{2x^4}{7} \div \frac{5x^4}{4}$$

$$= \frac{2x^4}{7} \cdot \frac{4}{5x^4}$$

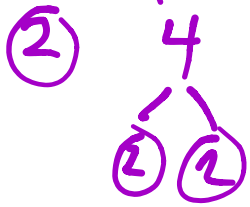
$$= \frac{8x^4}{35x^4}$$

$$= \frac{8}{35}(1)$$

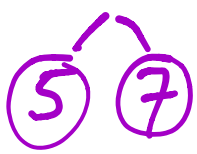
$$= \frac{8}{35}$$

Prime Factorization

$8 = 2^3$



$35 = 5 \cdot 7$



$$\frac{(a^2-4)}{(a^2-1)} \cdot \frac{(a+1)}{(a+2)} =$$

* cannot simplify a or a^2 b/c of + or − right now.

Recall $a^2-4 = (a-2)(a+2)$ difference of squares

$a^2-1 = (a-1)(a+1)$

$$= \frac{(a-2)\cancel{(a+2)}}{(a-1)\cancel{(a+1)}} \cdot \frac{\cancel{(a+1)}}{\cancel{(a+2)}}$$

$$= \frac{a-2}{a-1}$$

answer

* cannot simplify $\frac{a}{a}$ because "−" signs

Let $a=4$ $\quad \frac{(4)-2}{(4)-1} = \frac{2}{3}$

can't be simplified

$a=5$ $\quad \frac{5-2}{5-1} = \frac{3}{4}$

$\frac{2}{3} \neq \frac{3}{4} \rightarrow$ * cannot simplify $\frac{a}{a}$

$$\frac{x^2-36}{4x-x^2} \div \frac{3x-18}{x^2-8x+16}$$

$$\frac{(x^2-36)}{(4x-x^2)} \cdot \frac{(x^2-8x+16)}{(3x-18)}$$

$$\frac{(x-6)(x+6)}{x(4-x)} \cdot \frac{(x-4)(x-4)}{(3)(x-6)}$$

$$\frac{(x+6)(x-4)(x-4)}{3x(-1)(x-4)}$$

$$= \frac{(x+6)(x-4)}{-3x}$$

$$= -\frac{(x+6)(x-4)}{3x}$$

Recall

$x^2-8x+16$

$a = 1$

$b = -8$

$c = 16$

$ac = (1)(16) = 16 = -4 \cdot -4$

$b = \quad = -8 = -4 + -4$

$a-b = -(b-a)$

eg.

$5-3 = 2$

$3-5 = -2$

$-1(3-5) = 2$

$$\frac{3x^2y}{2ab} \cdot \frac{14a^2b}{18xy^2} =$$

$$= \frac{3x^2y}{18xy^2} \cdot \frac{14a^2b}{2ab}$$

$$= \frac{1x}{6y} \cdot \frac{7a}{1}$$

$$= \frac{7ax}{6y}$$

Addition & Subtraction

$$\frac{5}{21y} + \frac{4}{21y} = \frac{5+4}{21y} = \frac{9}{21y}$$

$$\frac{3x+11}{x+2} + \frac{x-3}{x+2} = \frac{3x+11+x-3}{x+2} = \frac{4x+8}{x+2}$$

$$= \frac{4(x+2)}{(x+2)}$$

$$= 4$$

* For common denominators

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{x^2}{x-5} - \frac{(10x-25)}{x-5} =$$

$$\frac{x^2-(10x-25)}{x-5} =$$

$$= \frac{x^2-10x+25}{x-5}$$

$$= \frac{(x-5)(x-5)}{x-5}$$

$$= x-5$$

Factor

$x^2-10x+25$

$a=1$

$b=-10$

$c=25$

$ac=25 = -5 \cdot -5$

$b=-10 = -5+-5$

$$\frac{6}{5p} - \frac{7}{2p^2}$$

* Need to find least common denominator.

LCD of $5p$ & $2p^2 = 10p^2$

$$\left(\frac{LCD}{LCD}\right)\frac{6}{5p} - \frac{7}{2p^2}\left(\frac{LCD}{LCD}\right)$$

* smallest number our constants multiply into
* largest power of variables

$$\left(\frac{10p^2}{10p^2}\right)\left(\frac{6}{5p}\right) - \left(\frac{7}{2p^2}\right)\left(\frac{10p^2}{10p^2}\right)$$

→ the largest power of each prime constant factor & variable

$$\frac{2p \cdot 6}{10p^2} - \frac{5 \cdot 7}{10p^2}$$

$$\frac{12p - 35}{10p^2}$$

* $\frac{a}{b} + \frac{c}{d}$

CD of $bd = bd$

$$= \left(\frac{bd}{bd}\right)\frac{a}{b} + \frac{c}{d}\left(\frac{bd}{bd}\right)$$

$$= \frac{ad + bc}{bd}$$

$$\frac{4a-20}{3a-15} - \frac{a-5}{a-5}$$

must be LCM

LCD of $3a-15$ & $a-5$:

$3 \cdot (a-5)$

$(a-5)$

$$\frac{4a-20}{3(a-5)} - \frac{a-5}{a-5}$$

$$\left(\frac{3(a-5)}{3(a-5)}\right)\frac{4a-20}{3(a-5)} - \frac{a-5}{a-5}\left(\frac{3(a-5)}{3(a-5)}\right)$$

LCD: $3^1 \cdot (a-5)^1$

$3(a-5)$

* Recall PEMDAS.
Multiplication/Division before addition/subtraction

$$\frac{4a-20}{3(a-5)} - \frac{3(a-5)}{3(a-5)}$$

$$\frac{4a-20}{3(a-5)} - \frac{3a-15}{3(a-5)}$$

$$\frac{(4a-20)-(3a-15)}{3(a-5)}$$

$$= \frac{4a-20+(-3a)+15}{3(a-5)}$$

$$= \frac{a-5}{3(a-5)}$$

$$= \frac{1\cdot(a-5)}{3\cdot(a-5)} = \frac{1}{3}$$

Note: $\frac{a-5}{a-5}$ is not correct.

$\frac{a-5}{a-5}$ is correct

$$\frac{y-4}{y-2} - \frac{y+4}{2-y}$$

Recall

$$a-b = -(b-a)$$

$$\frac{y-4}{y-2} - \frac{y+4}{-(y-2)}$$

LCM/LCD: $-(y-2)$

$$\frac{-(y-2)}{-(y-2)}\cdot\frac{y-4}{(y-2)} - \frac{y+4}{-(y-2)}\cdot\frac{-(y-2)}{-(y-2)}$$

$$= \frac{-(y-4) - (y+4)}{-(y-2)}$$

$$= \frac{-y+4 \; -y-4}{-(y-2)}$$

$$= \frac{-2y}{-(y-2)}$$

$$= \frac{2y}{y-2}$$

$$\frac{w-3}{w-4} - \frac{10w-32}{w^2-16}$$

$$\frac{w-3}{w-4} - \frac{10w-32}{(w-4)(w+4)}$$

LCD: $(w-4)(w+4)$

$$\frac{(w-4)(w+4)}{(w-4)(w+4)}\cdot\frac{(w-3)}{(w-4)} - \frac{(10w-32)}{(w-4)(w+4)}\cdot\frac{(w-4)(w+4)}{(w-4)(w+4)}$$

$$\frac{(w+4)(w-3) - (10w-32)}{(w-4)(w+4)}$$

$$\frac{w^2-3w+4w-12-10w+32}{(w-4)(w+4)}$$

$$\frac{w^2-9w+20}{(w-4)(w+4)}$$

$$\frac{(w-4)(w-5)}{(w-4)(w+4)} = \frac{w-5}{w+4}$$