Multiplication of radicals

$$\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n} \cdot b^{1/n}$$
$$= \sqrt[n]{a} \cdot \sqrt[n]{b}$$

index value

radicand

Simplified form of a radical

1) Radicand has no factor raised to a power greater than / equal to the index.
   ex: $\sqrt[2]{x^4} = x^2$

2) Radicand does not contain a fraction. $\leftarrow \sqrt{\frac{1}{4}}$

3) No radicals in the denominator of a fraction.
   ex: $\frac{2}{\sqrt{2}}$

Ex 1: Simplify the following.

- $\sqrt[2]{x^6} = x^3$
- $\sqrt{x^{100}} = x^{50}$
- $\sqrt[3]{x^6} = \sqrt[3]{x^2} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{x^2}$

$\rightarrow x^2$

Ex 2: Simplify $\sqrt{56}$

a) $\sqrt{56} = \sqrt{4} \cdot \sqrt{14}$
$= \boxed{2 \cdot \sqrt{14}}$

$$\begin{array}{r|l} 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array}$$

$56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

$\sqrt{56} = \sqrt{4} \cdot \sqrt{14}$

$1^2 \rightarrow 1$
$2^2 \rightarrow 4$
$3 \rightarrow 9$
$4 \rightarrow 16$
$5 \rightarrow 25$
$6 \rightarrow 36$
$7 \rightarrow 49$
$8 \rightarrow 64$
$9 \rightarrow 81$
$10 \rightarrow 100$
$11 \rightarrow 121$
$12 \rightarrow 144$
$13 \rightarrow 169$

b) $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$
$= 2\sqrt{6}$

$$\begin{array}{r|l} & 24 \\ 1 & 24 \\ 3 & 8 \\ 4 & 6 \end{array}$$

c) $6\sqrt{50}$
$= 6\sqrt{25}\sqrt{2}$
$= 6 \cdot 5 \cdot \sqrt{2}$
$= 30\sqrt{2}$

d) $\sqrt{35} = \sqrt{35}$
$= \sqrt{7} \cdot \sqrt{5}$

$$\begin{array}{r|l} & 35 \\ 1 & 35 \\ 7 & 5 \end{array}$$

e) $\frac{7\sqrt{50}}{10} = \frac{7\sqrt{25}\sqrt{2}}{10}$
$= \frac{7 \cdot \cancel{5}^{1} \sqrt{2}}{\cancel{10}_{2}} \quad = \frac{35\sqrt{2}}{10}$
$= \frac{7\sqrt{2}}{2}$

You try!

$$\sqrt{54a^4b^3} = \sqrt{9}\sqrt{6}\cdot a^2\sqrt{b^3}$$
$$= 3\sqrt{6}\,a^2\sqrt{b^2}\sqrt{b}$$
$$= 3\sqrt{6}\,a^2 b\sqrt{b}$$
$$= \boxed{3a^2b\sqrt{6b}}$$

$b^3 = b^2 \cdot b^1$
$= b^{2+1} = b^3$

Recall: $\sqrt[n]{a^n} = a$
ex: $\sqrt[3]{a^3} = a$

Add / subtract radicals

$2x + 3x = 5x$ $\quad x \cdot x = x^2$
$x + x = 2x$

"Like" radicals have terms with the same index and same radicand.

a) $3\sqrt{x} + 7\sqrt{x} = 10\sqrt{x}$

b) $\boxed{3\sqrt{x} + 7\sqrt{2x}}$

c) $1\sqrt{3} + 1\sqrt{3} = 2\sqrt{3}$ ⟶ $\sqrt{x} + \sqrt{y} \neq \sqrt{x+y}$
$\sqrt{3} + \sqrt{4} \neq \sqrt{7}$

d) $6\sqrt{11} - 2\sqrt{11} = 4\sqrt{11}$

e) $\frac{1}{4}x\sqrt{3y} - \frac{3}{2}x\sqrt{3y}$
$$= \left(\frac{1}{4} - \frac{3\cdot 2}{2\cdot 2}\right)x\sqrt{3y}$$
$$= \left(\frac{1}{4} - \frac{6}{4}\right)x\sqrt{3y}$$
$$= -\frac{5}{4}x\sqrt{3y}$$

Ex 4

a) $3\sqrt{8} + \sqrt{2}$
$$= 3\sqrt{4}\sqrt{2} + \sqrt{2}$$
$$= 6\sqrt{2} + 1\sqrt{2} = 7\sqrt{2}$$

b) $8\sqrt{x^3y^2} - 3y\sqrt{x^3}$
$$= 8\sqrt{x^2}\sqrt{x}\sqrt{y^2} - 3y\sqrt{x^2}\sqrt{x}$$
$$= 8\cdot x\sqrt{x}\,y - 3y\cdot x\sqrt{x}$$
$$= 8xy\sqrt{x} - 3xy\sqrt{x}$$
$$= \boxed{5xy\sqrt{x}}$$

$2\cdot 3 = 3\cdot 2$
$2+3 = 3+2$
$2-3 \neq 3-2$

$\sqrt{y^3} = \sqrt{y^2}\sqrt{y} = y\sqrt{y}$

c) $\sqrt{50x^2y^5} - 13y\sqrt{2x^2y^3} + xy\sqrt{98y^3}$

$= \sqrt{25 \cdot 2 x^2 y^5} - 13y x\sqrt{2y} + xy\sqrt{49 \cdot 2y^3}$

$= 5xy^2\sqrt{2y} - 13y^2x\sqrt{2y} + 7xy^2\sqrt{2y}$

$= \boxed{-xy^2\sqrt{2y}}$

$\sqrt{y^5} = \sqrt{y^4}\cdot\sqrt{y^1} = y^2$

$\sqrt{y^{23}} = \sqrt{y^{22}}\cdot\sqrt{y^1}$

$= y^{11}\cdot\sqrt{y}$

d) Add $3\sqrt{2x^2} + \sqrt{8}$

$\boxed{3x\sqrt{2} + 2\sqrt{2}}$

$\sqrt{8}$

$\sqrt{4}\ \ \sqrt{2}$

$= 2\sqrt{2}$

You try!

$4\sqrt{45} - \sqrt{5y^4}$

$= 4\cdot\underbrace{\sqrt{9}}_{3}\cdot\sqrt{5} - y^2\sqrt{5}$

$= \boxed{12\sqrt{5} - y^2\sqrt{5}}$

Multiplication property of radicals.

$$\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$$

Ex1: Mutiply the expression.

a) $(3\sqrt{2})(5\sqrt{6})$

$= (3\cdot 5)(\sqrt{2}\cdot\sqrt{6})$

$= 15\sqrt{12}$

$= 15\sqrt{4}\sqrt{3}$

$= \boxed{30\sqrt{3}}$

b) $(2x\sqrt{y})(-7\sqrt{xy})$

$= (2x \cdot -7)(\sqrt{y}\cdot\sqrt{xy})$

$= \boxed{-14xy\sqrt{x}}$

$\sqrt{xy^2} = y\sqrt{x}$

c) $3\sqrt{11}\,(2+\sqrt{11})$

$= 6\sqrt{11} + 3\cdot 11$

$= \boxed{6\sqrt{11} + 33}$

d) $(\sqrt{5}+3\sqrt{2})(2\sqrt{5}-\sqrt{2})$

$= 10 - \sqrt{10} + 6\sqrt{10} - 6$

$= \boxed{4+5\sqrt{10}}$

$2x(3+7y)$

$\sqrt[n]{a^n} = a$

Special case products

- $(a+b)^2 = (a+b)(a+b)$
  $= a^2 + 2ab + b^2$
- $(a-b)^2 = (a-b)(a-b)$
  $= a^2 - ab - ab + b^2$
  $= a^2 - 2ab + b^2$

Ex 2: Square the radical expression.

a) $(\sqrt{d}+3)^2$

$= (\sqrt{d}+3)(\sqrt{d}+3)$

$= d + 3\sqrt{d} + 3\sqrt{d} + 9$

$= \boxed{d + 6\sqrt{d} + 9}$

b.) $(5\sqrt{y} - \sqrt{2})^2$

$= (5\sqrt{y} - \sqrt{2})(5\sqrt{y} - \sqrt{2})$

$= 25y - 5\sqrt{2y} - 5\sqrt{2y} + 2$

$= \boxed{25y - 10\sqrt{2y} + 2}$

c.) $(\sqrt{3}+2)(\sqrt{3}-2)$

$(\sqrt{3})^2 - (2)^2$

$3 - 4 = \boxed{-1}$

$(a+b)(a-b) = a^2 - b^2$

* With radicals

$a^2 - b^2 =$ constant.

Division of Radicals

$$\sqrt[2]{\frac{a^6}{b^4}} = \frac{\sqrt{a^6}}{\sqrt{b^4}} = \frac{a^{\frac{6}{2}}}{b^{\frac{4}{2}}} = \frac{a^3}{b^2}$$

$$\sqrt{\frac{x^4}{y^{10}}} = \frac{x^{\frac{4}{2}}}{y^{\frac{10}{2}}} = \frac{x^2}{y^5}$$

Rationalize Denominators

a.) $$\sqrt{\frac{y^5}{7}} = \frac{\sqrt{y^5}}{\sqrt{7}} = \frac{\sqrt{y^4}\sqrt{y}}{\sqrt{7}} = \frac{y^2\sqrt{y}}{\sqrt{7}} = \frac{y^2\sqrt{y}}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right) = \boxed{\frac{y^2\sqrt{7y}}{7}}$$

* $\sqrt{7} \approx 2.6457\ldots$

* We rationalize because it's easier to divide by an integer, in this case, 7.

b.) $$\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}} = \frac{\sqrt{4}\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \boxed{\frac{2\sqrt{6}}{3}}$$

c.) $$\frac{4}{2-\sqrt{5}} = \frac{4}{2-\sqrt{5}}\left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right) = \frac{8+4\sqrt{5}}{4-5} = \frac{8+4\sqrt{5}}{-1} = \boxed{-8-4\sqrt{5}}$$

Multiply numerator and denominator by conjugate of denominator

Conjugates: $a-b$, $a+b$ because

$$(a-b)(a+b) = a^2+ab-ab-b^2$$
$$= a^2-b^2$$

d.) $$\frac{5}{10+\sqrt{7}} = \frac{5}{10+\sqrt{7}}\left(\frac{10-\sqrt{7}}{10-\sqrt{7}}\right) = \frac{50-5\sqrt{7}}{(10)^2-(\sqrt{7})^2} = \frac{50-5\sqrt{7}}{100-7} = \boxed{\frac{50-5\sqrt{7}}{93}}$$

e.) $$\frac{7\sqrt{14}}{7-\sqrt{14}} = \left(\frac{7\sqrt{14}}{7-\sqrt{14}}\right)\left(\frac{7+\sqrt{14}}{7+\sqrt{14}}\right) = \frac{49\sqrt{14}+7(\sqrt{14})^2}{(7)^2-(\sqrt{14})^2} = \frac{49\sqrt{14}+98}{49-14} = \frac{49\sqrt{14}+98}{35}$$

$$= \frac{\cancel{7}(7\sqrt{14}+14)}{\cancel{7}\cdot 5}$$

$$= \boxed{\frac{7\sqrt{14}+14}{5}}$$

f.) $\dfrac{x-5}{\sqrt{x}-\sqrt{5}} = \dfrac{x-5}{\sqrt{x}-\sqrt{5}}\left(\dfrac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}\right)$

$= \dfrac{\cancel{(x-5)}(\sqrt{x}+\sqrt{5})}{\cancel{(x-5)}} = \boxed{\sqrt{x}+\sqrt{5}}$

$*\ (\sqrt{x}+\sqrt{5})(\sqrt{x}-\sqrt{5}) = (\sqrt{x})^2-(\sqrt{5})^2$

$= x-5$

$(a+b)(a-b) = a^2-b^2$

$\rightarrow$ $\sqrt{x}-\sqrt{5}$ and $\sqrt{x}+\sqrt{5}$ are factors of $x-5$

$\rightarrow$ In general $a-b = (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$