

Complex Fractions

$$\frac{\frac{5x^2}{y}}{\frac{10x}{y^2}} = \frac{5x^2}{y} \div \frac{10x}{y^2} \quad \text{KCF}$$
$$= \frac{5x^2}{y} \cdot \frac{y^2}{10x}$$
$$= \frac{5x^2 y^2}{10xy} = \frac{5xy}{10} = \boxed{\frac{xy}{2}}$$

$$\left(\frac{4x}{4x}\right) \frac{1}{4x} - \frac{3}{2} \left(\frac{4x}{4x}\right)$$

$$\left(\frac{2x}{2x}\right) \frac{3}{1} - \frac{1}{2x} \left(\frac{2x}{2x}\right)$$

LCD numerator
 $4x$ and $2 = 4x$
 $2^1 x$ and $2^2 = 2^2 x$
 LCD: highest power of 2 and highest power of x.

LCD denominator:
 1 and $2x = 2x$
 highest power of 1 = 1
 " " " 2 = 1
 " " " x = 1
 $1 \cdot 2^1 \cdot x^1 = 2x$

Numerator: $\left(\frac{4x}{4x}\right) \frac{1}{4x} - \frac{3}{2} \left(\frac{4x}{4x}\right) = \frac{1 \cdot 1 - 3(2x)}{4x}$
 $= \frac{1 - 6x}{4x}$

Denominator: $\left(\frac{2x}{2x}\right) \frac{3}{1} - \frac{1}{2x} \left(\frac{2x}{2x}\right) = \frac{(2x)3 - 1}{2x}$
 $= \frac{6x - 1}{2x}$

$$\frac{\frac{1-6x}{4x}}{\frac{6x-1}{2x}} = \frac{\cancel{1-6x}^{-1}}{2 \cancel{4x}} \cdot \frac{\cancel{2x}^1}{(\cancel{6x-1})_1} = \frac{-1 \cdot 1}{1 \cdot 2} = -\frac{1}{2}$$

KCF

$$a - b = -(b - a)$$

$$\frac{1-6x}{6x-1} = \frac{-\cancel{(6x-1)}}{\cancel{(6x-1)}} = \frac{-1}{1} = -1$$

Method 1 for Simplifying Complex Fractions

1. **Combine** numerators & denominators separately

2. **Divide** resulting Fractions

- KCF

$$\frac{4 - \frac{6}{x}}{\frac{2}{x} - \frac{3}{x^2}}$$

LCD of every fraction

$$1, x, x, x^2$$

highest power of 1 = 1
 " " " x = 2

$$1 \cdot x^2 = x^2$$

$$= \frac{\left(4 - \frac{6}{x}\right) \left(\frac{\text{LCD}}{\text{LCD}}\right)}{\left(\frac{2}{x} - \frac{3}{x^2}\right) \left(\frac{\text{LCD}}{\text{LCD}}\right)}$$

$$= \frac{\left(4 - \frac{6}{x}\right) \cdot x^2}{\left(\frac{2}{x} - \frac{3}{x^2}\right) \cdot x^2}$$

$$\left(\frac{2}{x} - \frac{3}{x^2}\right) \cdot x^2$$

$$\frac{x^2}{1} \left(\frac{2}{x} - \frac{3}{x^2}\right)$$

$$\frac{2x^2}{1x} - \frac{3x^2}{1x^2}$$

$$= \frac{4x^2 - \frac{6x^2}{x}}{\frac{2x^2}{x} - \frac{3x^2}{x^2}}$$

$$= \frac{4x^2 - 6x}{2x - 3} = \frac{2x(2x - 3)}{(2x - 3)} = 2x$$

$$\frac{\left(y - \frac{1}{y}\right) y^2}{\left(1 - \frac{1}{y^2}\right) y^2}$$

$$= \frac{y^3 - 1 \frac{y^2}{y}}{1 \frac{y^2}{1} - 1 \frac{y^2}{y^2}}$$

LCD: y^2 (highest power of y)

$y^3 - y \neq y^2$
Not like terms

$$= \frac{y^3 - y}{y^2 - 1} = \frac{y \cancel{(y^2 - 1)}}{\cancel{(y^2 - 1)}} = y$$

Method 2 for Simplifying Complex Fractions

1. Find LCD for all fractions
2. Multiply both numerator & denominator by LCD
3. Simplify.

$$\frac{x^{-1} - x^{-2}}{1 + 2x^{-1} - 3x^{-2}}$$

$$\frac{\frac{1}{w+3} - \frac{1}{w-3}}{1 + \frac{9}{w^2-9}}$$

$$\frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$