$ax^2+bx+c=0$, $a,b,c \in \mathbb{R}$, $a \neq 0$

quadratic equation

$\mathbb{R}$ - real numbers

If $m \cdot n = 0$, then $m=0$ or $n=0$

$x(x-2)=0$

$x=0$ or $x-2=0$

$+2 \quad +2$

$x=2$

$x \in \{0,2\}$

↑ is one of the values in the set

$(x-4)(x+5)(x-\pi)=0$

$x-4=0$ or $x+5=0$ or $x-\pi=0$

$+4 \quad +4$ $\quad -5 \quad -5$

$x=4$ or $x=-5$ $\quad x=\pi$

$x \in \{-5, 4, \pi\}$

$x^2 - 6x + 8 = 0$ ← quadratic equation

polynomial equation

$a = 1 \quad b = -6 \quad c = 8$

Note degree of polynomial is 2, which is not 1. then we should put everything on one side.

$ac = (1)(8) = 8 = \underline{-4} \cdot \underline{-2}$

$b = -6 = \underline{-4} + \underline{-2}$

Only because $a = 1$

$(x-4)(x-2) = 0$

$x - 4 = 0$ or $x - 2 = 0$

$x \in \{2, 4\}$

$\boxed{x = 4 \text{ or } x = 2}$

$x^2 + 3x - 54 = 0$

$a = 1 \quad b = 3 \quad c = -54$

$ac = (1)(-54) = -54 = \underline{9} \cdot \underline{-6}$

$b = 3 = \underline{9} + \underline{-6}$

$x \in \{-9, 6\}$

$(x+9)(x-6) = 0$

$x + 9 = 0$ or $x - 6 = 0$

$\boxed{x = -9 \text{ or } x = 6}$

$2x^2+7x+6=0$

$a=2 \quad b=7 \quad c=6$

$ac=(2)(6)=12=3\cdot 4$

$b=7=3+4$

$2x^2+7x+6=0$

$2x^2+(3+4)x+6=0$

$2x^2+3x+4x+6=0$ ← Factor by Grouping

$(2x^2+3x)+(4x+6)=0$

$x\left(\frac{2}{x}x^2+\frac{3}{x}x\right)+2\left(\frac{4}{2}x+\frac{6}{2}\right)=0$

$x(2x+3)+2(2x+3)=0$

Notice $2x+3$ is our GCF.

let $2x+3=u$

$xu+2u=0$

$(x+2)u=0$

$(x+2)(2x+3)=0$

$x+2=0$ or $2x+3=0$

$-2\ -2 \qquad -3\ -3$

$x=-2 \qquad 2x=-3$

$x=-\frac{3}{2}$

$$x^3 - 7x^2 - 6x + 42 = 0$$

↖ polynomial equation

4 terms

try grouping

$$(x^3 - 7x^2) + (-6x + 42) = 0$$

$$x^2(x-7) + -6(x-7) = 0$$

$$(x^2-6)(x-7) = 0$$

$x^2 - 6 = 0$ or $x - 7 = 0$

$x^2 = 6$ $\quad$ $x = 7$

$x = \pm\sqrt{6}$

$$x^3 - 7x - 9x + 63 = 0$$

$$(x^3 - 7x) + (-9x + 63) = 0$$

$$x^2(x-7) + -9(x-7) = 0$$

$x^2 - 9 = 0$ or $x - 7 = 0$

$\boxed{x = 7}$

also factorable

$(x)^2 - (3)^2 = 0$

difference of squares

$x^2 - a^2 = (x+a)(x-a)$

$(x+3)(x-3) = 0$ $\quad x \in \{-3, 3, 7\}$

$x + 3 = 0$ or $x - 3 = 0$

$\boxed{x = -3}$ or $\boxed{x = 3}$

$x^2 - 8x + 16 = 0$

$a=1 \quad b=-8 \quad c=16$

$ac=(1)(16)=16=\underline{-4}\cdot\underline{-4}$

$b=-8=\underline{-4}+\underline{-4}$

$(x-4)(x-4)=0$

$x-4=0 \quad \text{or} \quad x-4=0$

$x=4$

$(x-4)^2=0$

$\sqrt{(x-4)^2}=\pm\sqrt{0}$

$x-4=0$

$x=4$

$x^2 - 8x + 16 = 0$

$a^2+2ab+b^2=(a+b)(a+b)$

$a^2-2ab+b^2=(a-b)(a-b)$

$(x)^2 - 2(4)x + (4)^2 = 0$

$(x-4)(x-4)=0$

$x-4=0 \quad \text{or} \quad x-4=0$

$\vdots$

$x=4$

$6x^2 = -8x$ ← still quadratic, largest power is 2

$+8x \quad +8x$ ← make one side = 0

$6x^2 + 8x = 0$

$2x(3x+4) = 0$

$x \in \{0, -\frac{4}{3}\}$

$\frac{2x}{2} = \frac{0}{2}$ or $3x+4=0$

$x = 0$

$-4 \quad -4$

$\frac{3x}{3} = \frac{-4}{3}$

$x = -\frac{4}{3}$

---

* Don't do this

$\frac{6x^2}{x} = \frac{-8x}{x}$

dividing by x prevented us from calculating x=0

$\frac{6x}{6} = \frac{-8}{6}$

* Don't divide both sides by variable

$x = -\frac{4}{3}$

* Whenever we have polynomial equation

$ax^n + bx^{n-1} + cx^{n-2} + \ldots = 0$

there should be n solutions.