Factoring by Grouping

Recall

$$(3a+2)(2b-7) = 3a(2b-7) + 2(2b-7)$$

by distributing $= 6ab - 21a + 4b - 14$

Our job is to "reverse the process"

$6ab - 21a + 4b - 14$

$(6ab - 21a) + (4b - 14)$

$3a(2b-7) + 2(2b-7)$

$(2b-7)(3a+2)$

$6ab + 4b - 21a - 14$

$(6ab + 4b) + (-21a - 14)$

$2b(3a+2) + (-7)(3a+2)$

$(2b + (-7))(3a+2)$

$(2b-7)(3a+2)$

Steps (* Find GCF of all terms first.)

1. Group 1st pair & second pair.
2. Factor the GCF of each pair
3. If both new terms share GCF, then factor it.

$x^3 + 3x^2 - 3x - 9$

$(x^3 + 3x^2) + (-3x - 9)$

$x^2(x+3) + (-3)(x+3)$

$(x+3)(x^2-3)$

$a^3 - 4a^2 - 3a + 12$

$(a^3 - 4a^2) + (-3a + 12)$

$a^2(a-4) + (-3)(a-4)$

$(a-4)(a^2-3)$

$24p^2q^2 - 18p^2q + 60pq^2 - 45pq \leftarrow$ GCF: $3pq$

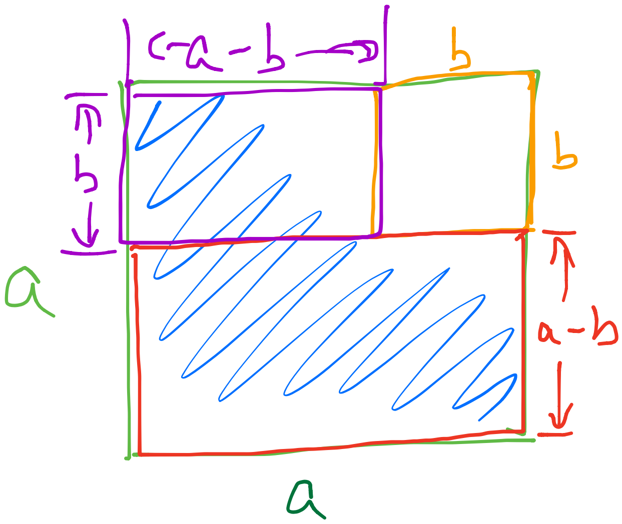
$(24p^2q^2 - 18p^2q) + (60pq^2 - 45pq)$

$6p^2q(4q-3) + 15pq(4q-3)$

$(6p^2q + 15pq)(4q-3)$

$3pq(2p+5)(4q-3)$

Difference of Squares "a^2-b^2"



Area of $\square = a^2$

Area of $\square = b^2$

Area of shaded region: a^2-b^2

Area of red rectangle: $a(a-b)$

Area of purple rectangle: $b(a-b)$

Sum of the areas $= a^2-b^2$

$$= a(a-b) + b(a-b)$$

$$= (a+b)(a-b)$$

$$\rightarrow \quad a^2-b^2 = (a+b)(a-b) \qquad *$$

$$a^2-b^2 \neq (a-b)(a-b)$$

DON'T DO THIS!

$$x^2 - 49 = (x)^2 - (7)^2 \qquad \sqrt{49} = 7$$

$$= (x+7)(x-7)$$

$$x^2 - 3 = (x)^2 - (\sqrt{3})^2 \qquad \sqrt{3} = \sqrt{3}$$

$$= (x+\sqrt{3})(x-\sqrt{3})$$

$$98c^2d - 50d^3$$

$$= 2d(49c^2 - 25d^2)$$

$$= 2d((7c)^2 - (5d)^2)$$

$$= 2d(7c+5d)(7c-5d)$$

$$z^4 - 81$$

$$= (z^2)^2 - (9)^2$$

$$= (z^2 - 9)(z^2 + 9)$$

$$= (z-3)(z+3)(z^2+9)$$

Recall $z^4 = z \cdot z \cdot z \cdot z$

$= (z \cdot z)(z \cdot z)$

$= (z^2)^2$

Note: we cannot factor $a^2 + b^2$ at this point.

$$z^2 + 9 \neq (z+3)(z+3)$$

"Freshman Dream"