Systems of Linear Equations

$x + y = 16$

$x - y = 4$

Option 1.

Substitution

$\frac{-y}{-1} = \frac{4-x}{-1}$

$y = \frac{4}{-1} - \frac{x}{-1}$

$y = -4 - (-x)$

$y = -4 + x$

$y = x - 4$

$x + (x-4) = 16$

$2x - 4 = 16$

$\quad +4 \;\; +4$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

$(10) - y = 4$

$-10 \qquad -10$

$-y = -6$

$y = 6$

Solution

$(x, y) = (10, 6)$

$$x + y = 16$$
$$x - y = 4$$

by elimination

1. Add equations

$$\begin{array}{rcl} x \cancel{+ y} & = & 16 \\ x \cancel{- y} & = & 4 \\ \hline 2x & = & 20 \\ \frac{2x}{2} & = & \frac{20}{2} \\ x & = & 10 \end{array}$$

Solved for x

2. Pick one of original equation

$$x + y = 16$$

$$\begin{array}{rcl} \cancel{(10)} + y & = & 16 \\ \cancel{-10} & & -10 \\ \hline y & = & 6 \end{array}$$

Solution:

$$(x, y) = (10, 6)$$

Q2. $x - 2y = 4$
$-2x + 4y = 6$

By elimination

0. Choose variable to eliminate
1. Multiply by factor to be able to cancel 1 variable
2. Add the two equations
3. Solve for one variable
4. Solve for other variable

$$2(x - 2y = 4)$$
$$1(-2x + 4y = 6)$$

$$\begin{aligned} \cancel{2x} - \cancel{4y} &= 8 \\ -\cancel{2x} + \cancel{4y} &= 6 \\ \hline 0 &= 14 \end{aligned}$$

implies no solution
contradiction $0 \neq 14$

$x - 2y = 4$ and $-2x + 4y = 6$ are parallel lines

Q3. $1(-2x+4y=6)$

$-2(-x+2y=3)$

1. Choose to eliminate x

$$\begin{array}{r} -\cancel{2x} + \cancel{4y} = 6 \\ \cancel{2x} - \cancel{4y} = -6 \\ \hline 0 = 0 \end{array}$$

→ true statement

infinite solutions

$-2x+4y=6$
$-x+2y=3$

they are the same line.

identity

3x3 systems

$$\begin{aligned} x + y + 3z &= 1 \quad (A) \\ -x - 2y + z &= 2 \quad (B) \\ 5x + y + 2z &= 17 \quad (C) \end{aligned}$$

1. Pick a variable to get rid of (choose z)

2. Pick 2 equations and eliminate z

$$\begin{aligned} (B)\quad -x - 2y + z &= 2 \\ (C)\quad 5x + y + 2z &= 17 \end{aligned}$$

$$\begin{aligned} -2(B)\quad 2x + 4y \cancel{- 2z} &= -4 \\ 1(C)\quad 5x + y + 2\cancel{z} &= 17 \\ \hline 7x + 5y &= 13 \quad (D) \end{aligned}$$

3. Pick another 2 equations and eliminate z.

$$\begin{aligned} (A)\quad x + y + 3z &= 1 \\ (B)\quad -x - 2y + z &= 2 \end{aligned}$$

$$1(A)\quad x + y + 3z = 1$$

$$-3\,B \quad 3x + 6y - 3z = -6$$

$$4x + 7y = -5 \qquad E$$

4. Solve the system of 2 equations D and E

$$-4\,D \quad 7x + 5y = 13$$

$$7\,E \quad 4x + 7y = -5$$

Eliminate x by choice.

$$D \quad -28x - 20y = -52$$

$$E \quad 28x + 49y = -35$$

$$\frac{29y}{29} = \frac{-87}{29}$$

$$y = -3$$

$$D \quad 7x + 5y = 13$$

$$7x + 5(-3) = 13$$

$$7x - 15 = 13$$

$$+15 \quad +15$$

$$\frac{7x}{7} = \frac{28}{7}$$

$$x = 4$$

5. Solve for final variable.

(A) $x + y + 3z = 1$

$(4) + (-3) + 3z = 1$

$\cancel{1} + 3z = 1$

$\cancel{-1} \quad -1$

$3z = 0$

$\frac{3z}{3} = \frac{0}{3}$

$z = 0$

Solution $(x, y, z) = (4, -3, 0)$