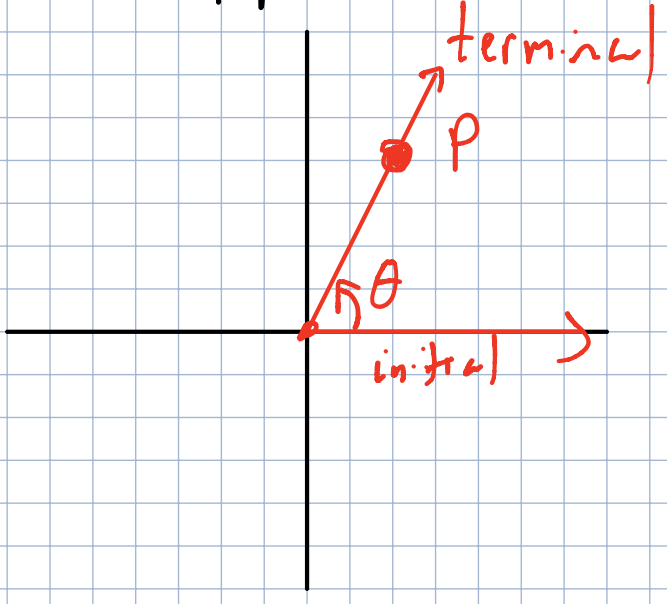
recall – initial side: fixed ray

terminal side: rotated ray

standard position – vertex is @ (0,0)

initial side → positive x-axis



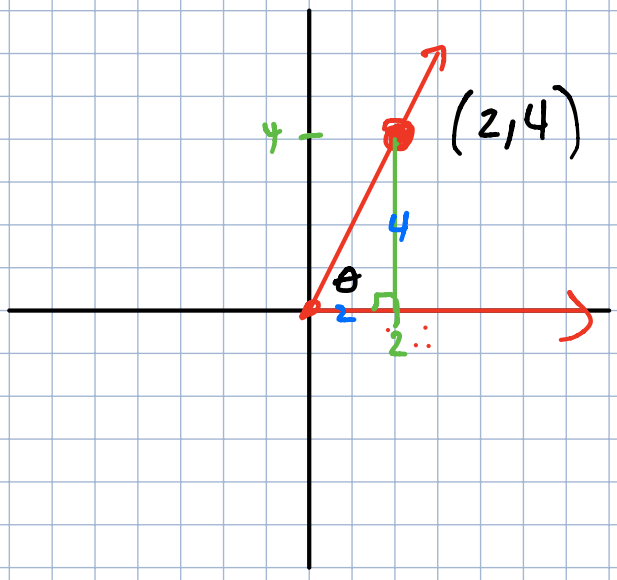
P is some point on terminal side.

Trig functions of any angle

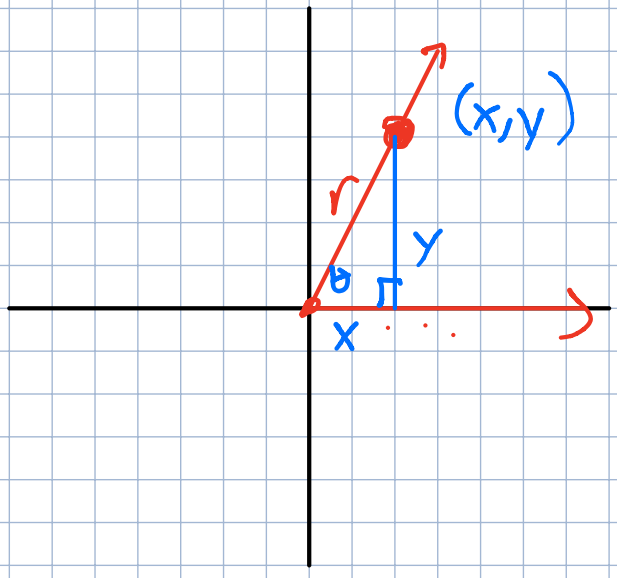
Given P(x,y) is any point on the terminal side of $\angle\theta$ in standard position.

The distance from the origin to P(x,y)

$r = \sqrt{x^2+y^2}$ → r is always positive.



Draw a line from the point that is perpendicular to x axis



x- adjacent

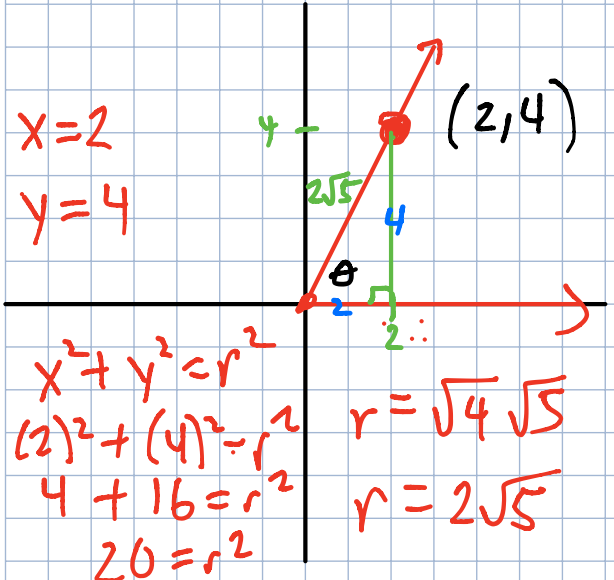
y- opposite

r- hypotenuse

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}, y \neq 0$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}, x \neq 0$

$\tan\theta = \frac{y}{x}, x \neq 0$ $\quad \cot\theta = \frac{x}{y}, y \neq 0$



$x = 2$

$y = 4$

$x^2 + y^2 = r^2$

$(2)^2 + (4)^2 = r^2$

$4 + 16 = r^2$

$20 = r^2$

$r = \sqrt{4}\sqrt{5}$

$r = 2\sqrt{5}$

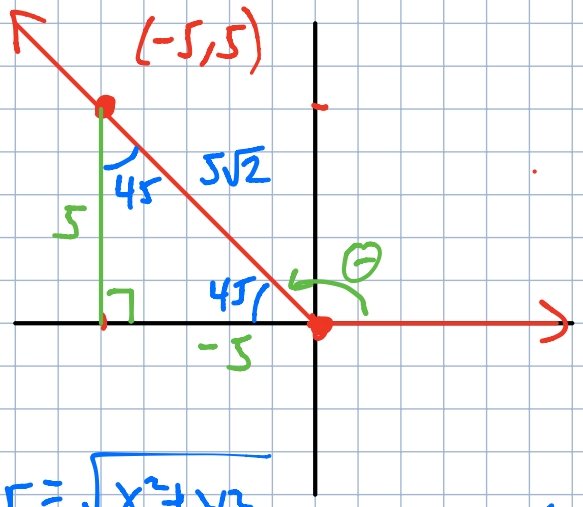
Given (2, 4)

$\sin\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \sec\theta = \frac{2\sqrt{5}}{2} = \sqrt{5}$

$\tan\theta = \frac{4}{2} = 2$ $\quad \cot\theta = \frac{2}{4} = \frac{1}{2}$

Find trig ratios of $P(-5,5)$



x = adjacent

y = opposite

r = hypotenuse

$x = -5$

$y = 5$

$r = 5\sqrt{2}$

Q2

(−, +)

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-5)^2+(5)^2}$

$r = \sqrt{25+25}$

$r = \sqrt{50}$

$r = \sqrt{25}\sqrt{2}$

$r = 5\sqrt{2}$

$\sin\theta = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

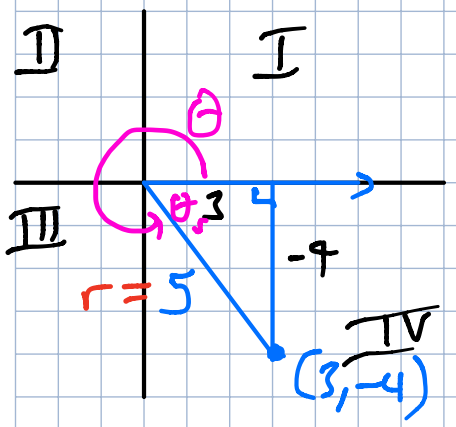
$\tan\theta = \frac{5}{-5} = -1$

$\csc\theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = -\frac{\sqrt{2}}{1} = -\sqrt{2}$

$\cot\theta = -1$

Given the point $(3,-4)$ Find six trig ratios



$x = 3$

$y = -4$

$r = 5$

Quadrant: 4

$x^2 + y^2 = r^2$

$(3)^2 + (-4)^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

$5 = r$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} = -\frac{4}{5}$

$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} = \frac{3}{5}$

$\sec\theta = \frac{r}{x} = \frac{5}{3}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{-4}{3}$

$\cot\theta = \frac{x}{y} = -\frac{3}{4} = -\frac{3}{4}$

* In reality θ is usually the angle in standard position
  - vertex is @ $(0,0)$
  - initial side is positive x-axis

* we use the reference angle θ_r
  - acute angle between terminal side and the x-axis

Observations about coordinate plane trigonometry

II (−,+)	I (+,+)
III (−,−)	IV (+,−)

Recall formulas

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Note: r is always positive

In QI, ALL trig ratios are positive.

In QII, $\sin\theta$ and $\csc\theta$ are positive.

In QIII, $\tan\theta$ and $\cot\theta$ are positive.

In QIV, $\cos\theta$ and $\sec\theta$ are positive.

Everything else is negative

Sinθ	All
Tanθ	Cosθ

ASTC

"All Students Take Calculus"

* only tells you positive

$\sin\theta = \frac{3}{11}$ and $\tan\theta < 0$

→ $\sin\theta$ is positive → QI, QII

→ $\tan\theta$ is negative → QII, QIV

Quadrant: 2 → $(-, +)$

$x = -4\sqrt{7}$

$y = 3$

$r = 11$

$x^2 + y^2 = r^2$

$x^2 + (3)^2 = (11)^2$

$x^2 + 9 = 121$

$-9 \quad -9$

$x^2 = 112$

$x = \pm\sqrt{112}$

$x = \pm\sqrt{16}\sqrt{7}$

$x = \pm 4\sqrt{7}$

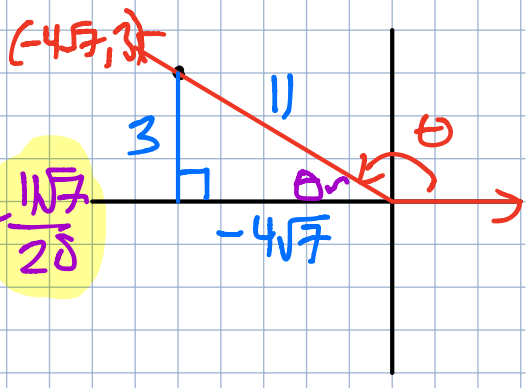
$x = -4\sqrt{7}$

$\sin\theta = \frac{3}{11}$ $\quad \csc\theta: \frac{11}{3}$

$\cos\theta = -\frac{4\sqrt{7}}{11}$ $\quad \sec\theta: -\frac{11}{4\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = -\frac{11\sqrt{7}}{28}$

$\tan\theta = -\frac{3}{4\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ $\quad \cot\theta: -\frac{4\sqrt{7}}{3}$

$= -\frac{3\sqrt{7}}{28}$



* $\tan\theta = \frac{24}{7}$ and $\cos\theta < 0$, Find 6 trig ratios

* $\tan\theta = \frac{24}{7}$ and $\cos\theta < 0$, Find 6 trig ratios

→ $\tan\theta$ is positive ∴ QI, QIII Quadrant III

→ $\cos\theta$ is negative: QII, QIII $(x,y) \Rightarrow (-,-)$

$x = -7$

$y = -24$

$r = 25$

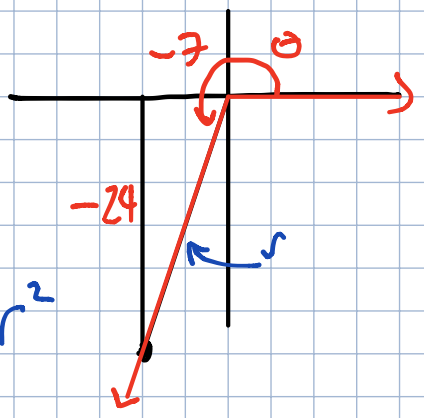
$x^2 + y^2 = r^2$

$(-7)^2 + (24)^2 = r^2$

$(49) + (576) = r^2$

$625 = r^2$

$25 = r$



Note: r is always positive

$\sin\theta: -\frac{24}{25}$

$\cos\theta: -\frac{7}{25}$

$\tan\theta: \frac{24}{7}$

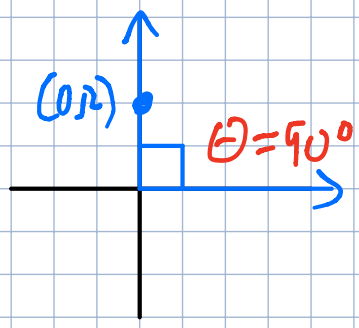
$\csc\theta: -\frac{25}{24}$

$\sec\theta: -\frac{25}{7}$

$\cot\theta: \frac{7}{24}$

Quadrantal Angles

Given: $P(0,2)$ ← could be any value on positive y-axis



$x = 0$

$y = 2$

$r = 2$

$\sin(90^\circ) \frac{2}{2} = 1$

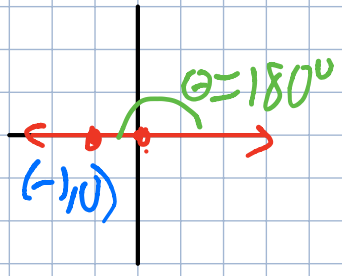
$\cos(90^\circ) \frac{0}{2} = 0$

$\tan(90^\circ) \frac{2}{0} = \text{undefined}$

$\csc(90^\circ) 1$

$\sec(90^\circ) \frac{2}{0} = \text{undefined}$

$\cot(90^\circ) \frac{0}{2} = 0$



$\sin\theta = 0$

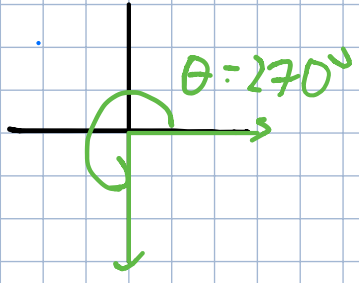
$\cos\theta = -1$

$\tan\theta = 0$

$\csc\theta = \text{undefined}$

$\sec\theta = -1$

$\cot\theta = \text{undefined}$



$\sin(270^\circ) = -1$

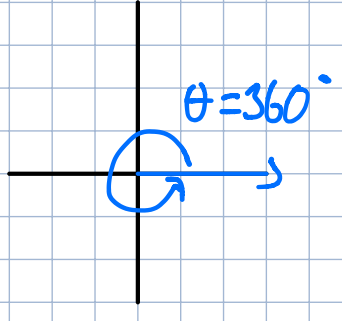
$\cos(270^\circ) = 0$

$\tan(270^\circ) = \text{undefined}$

$\csc(270) = -1$

$\sec(270) = \text{undef}$

$\cot(270) = 0$



θ deg	θ rad	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	0	1	0			
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$			
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1			
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$			
90	$\frac{\pi}{2}$	1	0	undefined			
180	π	0	-1	0			
270	$\frac{3\pi}{2}$	-1	0	undefined			
360	2π	0	1	0			