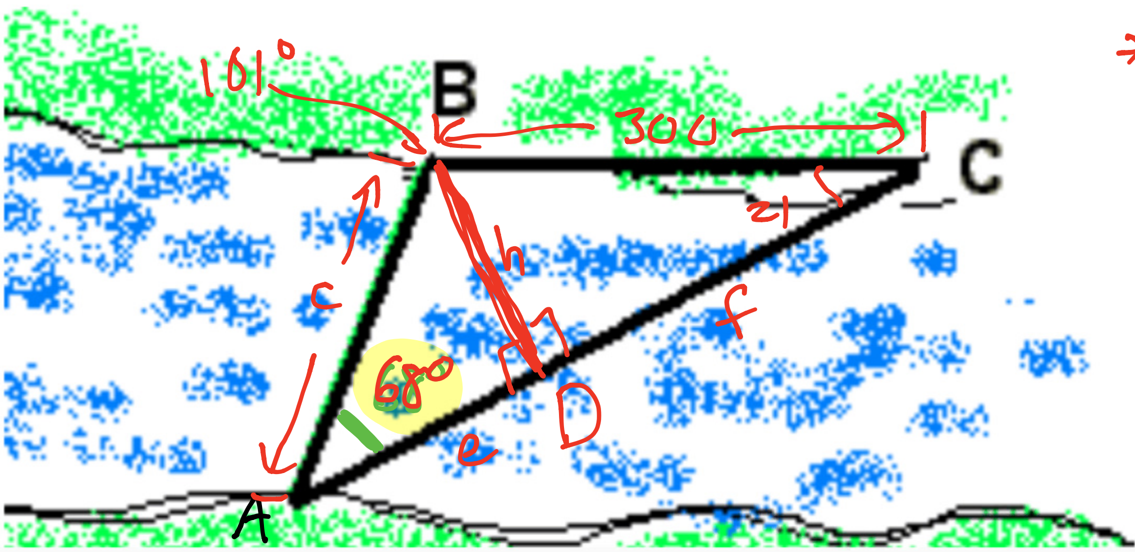
Recall from WW Trig Applications #1.

To find the distance AB across a river, a distance BC=300 is laid off on one side of the river. It is found that B=101° and C=21°. Find AB.




*Draw h such that $h \perp \overline{BC}$

not a right $\Delta$,

→ make right triangles

→ Looking for side c.

$m(\angle A) = 180 - (101 + 21)$

$= 180 - 122$

$= 68°$

$\Delta BCD$ is a right $\Delta$

$\Delta ABD$ is a right $\Delta$

$\sin(\angle C) = \frac{opp}{hyp}$

$\frac{\sin(21°)}{1} = \frac{h}{300}$

$h = 300\sin(21°)$

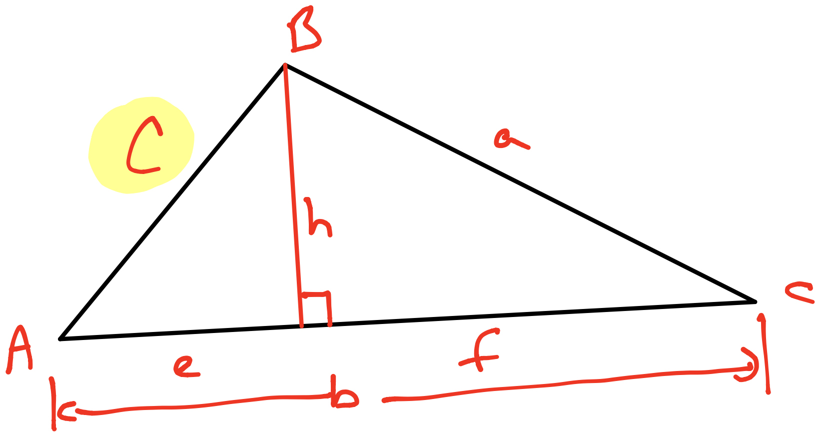
$\sin(\angle A) = \frac{opp}{hyp}$

$\frac{\sin(68°)}{1} = \frac{h}{c}$

$h = c\sin(68)$

$\frac{300\sin(21)}{\sin(68)} = \frac{c\,\cancel{\sin(68)}}{\cancel{\sin(68)}}$

$\boxed{115.95} \approx \frac{300\sin(21)}{\sin(68)} = c = AB$




Need to find side C.

$$\sin(\angle A) = \frac{opp}{hyp} \qquad \sin(\angle C) = \frac{opp}{hyp}$$

$$\sin(\angle A) = \frac{h}{c} \qquad \sin(\angle C) = \frac{h}{a}$$

$$h = c\sin(\angle A) \qquad h = a\sin(\angle C)$$

* Since both equations $= h$

$$\frac{\cancel{c}\cdot\sin(\angle A)}{\cancel{c}} = \frac{a\cdot\sin(\angle C)}{c}$$

$$\sin(\angle A) = \frac{a\sin(\angle C)}{c}$$

$$\frac{\sin(\angle A)}{a} = \frac{\cancel{a}\sin(\angle C)}{\cancel{a}\,c}$$

$$\frac{\sin(\angle A)}{a} = \frac{\sin(\angle C)}{c} = \frac{\sin(\angle B)}{b}$$

By similar logic

$\boxed{\text{* Law of sines}}$

Recall from WW Trig Applications #1.

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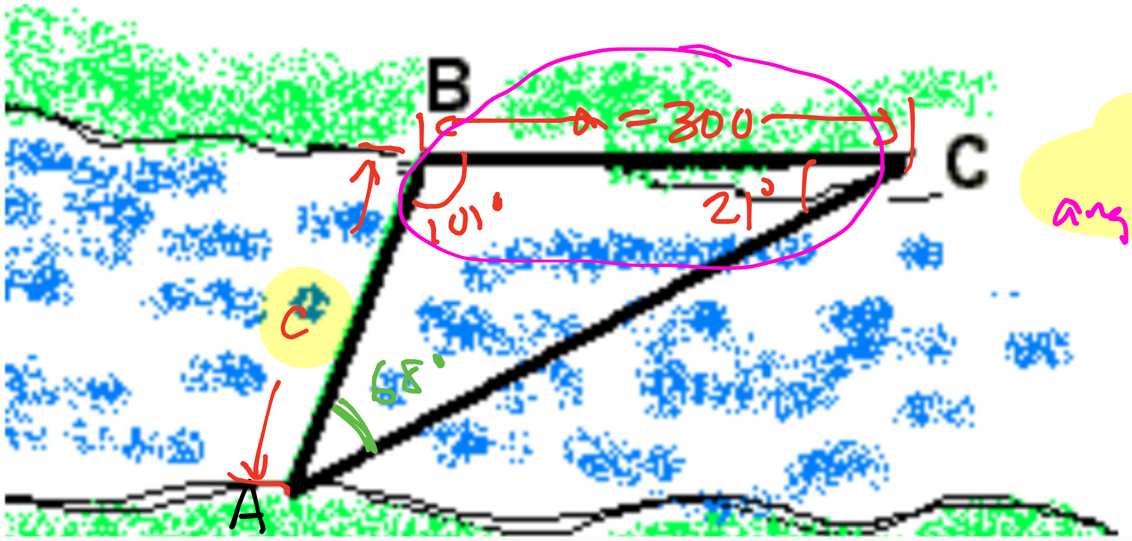



Given ASA case angle-side-angle

By Law of Sines

$$\frac{\sin(\angle C)}{c} = \frac{\sin(\angle A)}{a} = \frac{\sin(\angle A)}{BC}$$

$$\frac{\sin(21^\circ)}{c} = \frac{\sin(68^\circ)}{300}$$ cross multiplication

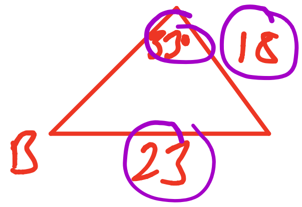
$$c\sin(68^\circ) = 300\sin(21^\circ)$$

$$c = \frac{300\sin(21)}{\sin(68)}$$

$$c \approx 115.95$$

* Law of sines is useful in if given ASA (angle side angle) or AAS (angle angle side) cases

$$\frac{\sin(33^\circ)}{23} = \frac{\sin(\angle B)}{18}$$




Side-side angle case

* cross multiply

$$\frac{18\sin(33)}{23} = \frac{\cancel{23}\sin(\angle B)}{\cancel{23}}$$

$$\frac{18\sin(33)}{23} = \sin(\angle B)$$

$$\arcsin\left(\frac{18\sin(33)}{23}\right) = \arcsin(\sin(\angle B))$$

(use calculator)

$$25{,}229^\circ \approx m(\angle B)$$

reference angle also

---

Recall $\sin(\angle B) = +\dfrac{18\sin(33)}{23}$

$\sin(\angle B)$ is positive in QI, QII

$0 < m(\angle B) < 90$

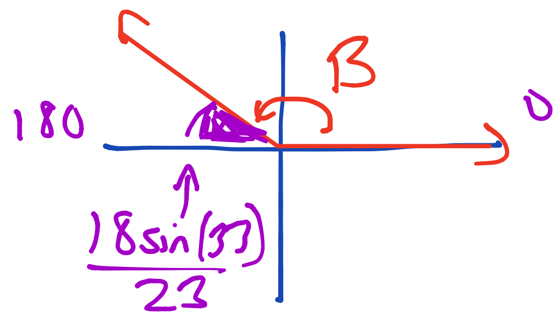
$90 < m(\angle B) < 180$

since $\sin(\angle B)$ is positive in QII also

ref $\angle B = \dfrac{18\sin(33)}{23}$

$m(\angle B) = 180 - \dfrac{18\sin(33)}{23}$

$\approx 154.771^\circ$




Check: we were given 33° as part of the triangle

Test: $154{,}771^\circ + 33^\circ < 180^\circ$

$187.771^\circ \not< 180^\circ$

reject $154{,}771^\circ$ not a solution

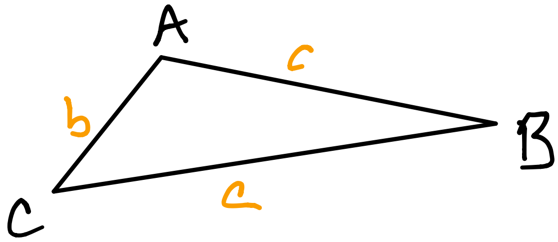
Note: 1. sum of interior angle measures of $\triangle = 180$
2. quadrant 2 ends at $180^\circ$

* Check both solutions if you are trying to the angle in the Law of sines.

In the SSA case, it is possible to have
0 solutions
1 solution
2 solutions

Check all calculated solutions

# Law of Cosines




$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$$\rightarrow a^2 = b^2 + c^2 - 2bc\cos(\angle A)$$

by similar logic

$$\rightarrow b^2 = a^2 + b^2 - 2ac\cos(\angle B)$$

* Use law of cosines in SAS or SSS case

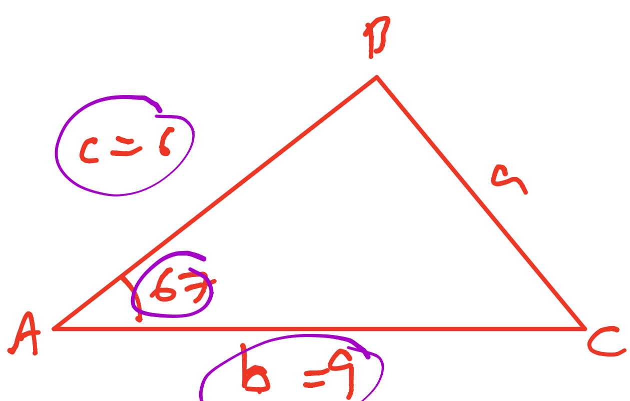
| SAS | SSS |
|---|---|
| side | side |
| angle | side |
| side | side |

## Q13c Final Exam Review Expanded

$b = 9$ $\quad$ $\angle A = 67^\circ$

$c = 6$

* Need to find side a




SAS case
Law of cosines

### Using Law of cosines

$$\rightarrow a^2 = b^2 + c^2 - 2bc\cos(\angle A)$$

$$a^2 = (9)^2 + (6)^2 - 2(9)(6)\cos(67^\circ)$$

$$a^2 = 81 + 36 - 108\cos(67)$$

$$a^2 = 117 - 108\cos(67)$$

$$a = \pm\sqrt{117 - 108\cos(67)}$$

$$a = \sqrt{117 - 108\cos(67)}$$

lengths are not negative.

$a \approx 8.649$ ← rounded answer.

## Secondary form to the Law of Cosines

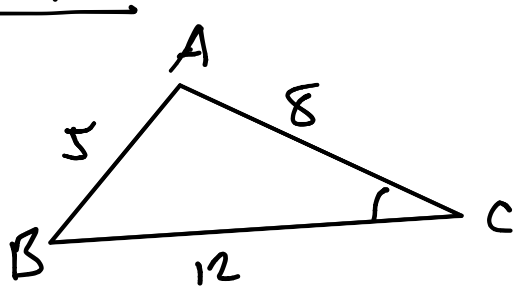
SSS form

$$\cos(\angle C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\rightarrow \cos(\angle A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\rightarrow \cos(\angle B) = \frac{a^2 + c^2 - b^2}{2ac}$$

Q13A $a = 12$ $b = 8$ $c = 5$, find $\angle C$




SSS case

$$\cos(\angle C) = \frac{a^2 + b^2 - c^2}{2ab}$$

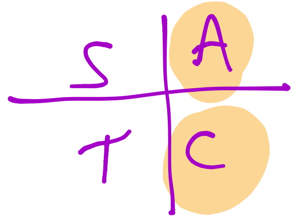
$$\cos(\angle C) = \frac{(12)^2 + (8)^2 - (5)^2}{2(12)(8)}$$

$$\cos(\angle C) = \frac{183}{192}$$

$$\arccos(\cos(\angle C)) = \arccos\left(\frac{183}{192}\right)$$

$$m(\angle C) = 17.612^\circ$$

Note $\cos(\angle C)$ is positive in QI and QIV




QI $0 < m(\angle C) < 90^\circ$

~~QIV $270^\circ < m(\angle C) < 360^\circ$~~

b/c we are looking for an angle of a triangle, we don't need to search a second answer.

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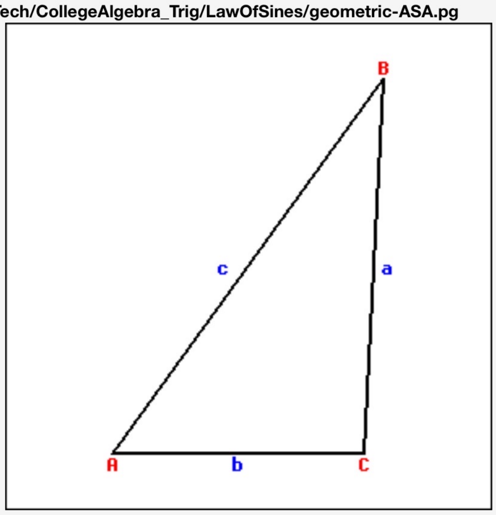
Previous Problem | Problem List | Next Problem

This set is visib

(1 point) **CUNY/CityTech/CollegeAlgebra_Trig/LawOfSines/geometric-ASA.pg**




Finish solving the triangle:

| | | | |
|---|---|---|---|
| $\angle A =$ | $54^\circ$ | a = | |
| $\angle B =$ | $33^\circ$ | b = | |
| $\angle C =$ | degrees | c = | 13 |

$$m(\angle C) = 180 - (54+33)$$

$$= 93^\circ$$

$$\frac{\sin(\angle C)}{c} = \frac{\sin(\angle A)}{a}$$

$$\frac{\sin(93)}{13} = \frac{\sin(54)}{a}$$

$$a\sin(93) = 13\sin(54)$$

$$a = \frac{13\sin(54)}{\sin(93)}$$

$$a \approx 10.532$$

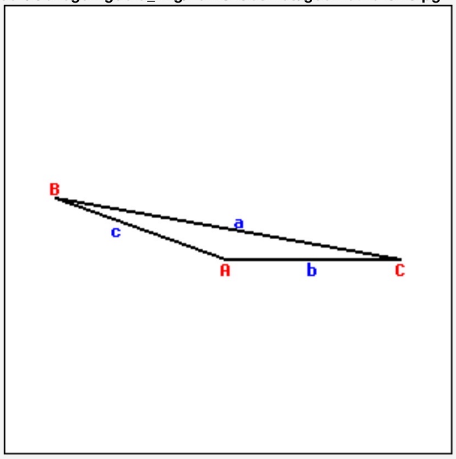
$$\frac{\sin(\angle C)}{c} = \frac{\sin(\angle B)}{b}$$

$$\frac{\sin(93)}{13} = \frac{\sin(33)}{b}$$

$$\vdots$$

$$b = \frac{13\sin(33)}{\sin(93)} \approx 6.898$$

(1 point) **CUNY/CityTech/CollegeAlgebra_Trig/LawOfCosines/geometric-SAS.pg**




Finish solving the triangle:

| | | | |
|---|---|---|---|
| $\angle A =$ | ☐ degrees | a = | 10 |
| $\angle B =$ | ☐ degrees | b = | 5 |
| $\angle C =$ | 10° | c = | ☐ |

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$$c^2 = (10)^2 + (5)^2 - 2(10)(5)\cos(10^\circ)$$

$$c^2 = 125 - 100\cos(10^\circ)$$

$$c = \sqrt{125 - 100\cos(10)}$$

$$c \approx 5.150$$

$$\cos(\angle B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(\angle B) = \frac{(10)^2 + \left(\sqrt{125 - 100\cos(10)}\right)^2 - (5)^2}{2(10)\left(\sqrt{125 - 100\cos(10)}\right)}$$

$$\cos(\angle B) = \frac{75 + 125 - 100\cos(10)}{20\sqrt{125 - 100\cos(10)}}$$

$$\cos(\angle B) = \frac{200 - 100\cos(10)}{20\sqrt{125 - 100\cos(10)}}$$

$$m(\angle B) = \arccos\left(\frac{10 - 5\cos(10)}{\sqrt{125 - 100\cos(10)}}\right) \approx 9.706$$

$$a^2 = b^2 + c^2 - 2bc\cos(\angle A)$$

$$(10)^2 = (5)^2 + \left(\sqrt{125 - 100\cos(10)}\right)^2 - 2(5)\sqrt{125-100\cos(10)}\cos(\angle A)$$

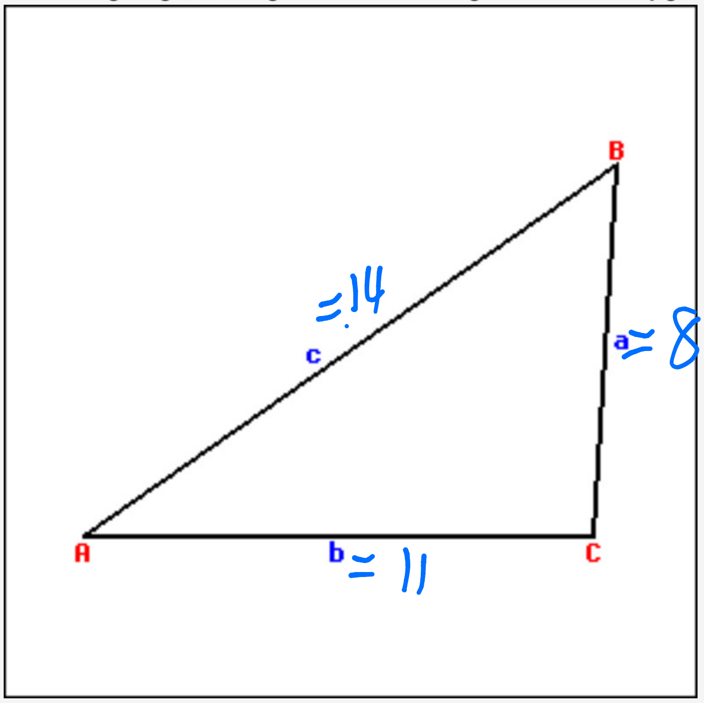
$$\frac{100 - (25 + 125 - 100\cos(10))}{-10\sqrt{125 - 100\cos(10)}} = \cos(\angle A)$$

$$\frac{-50 + 100\cos(10)}{-10\sqrt{125 - 100\cos(10)}} = \cos(\angle A)$$

$$\cos^{-1}\left(\frac{5 - 10\cos(10)}{\sqrt{125 - 100\cos(10)}}\right) = m(\angle A) \approx 160.294^\circ$$

* You can use original Law of Cosines also, ... just a bit of extra work.

(1 point) **CUNY/CityTech/CollegeAlgebra_Trig/LawOfCosines/geometric-SSS.pg**




SSS case

Finish solving the triangle:

| | | | |
|---|---|---|---|
| $\angle A =$ | ☐ degrees | a = | 8 |
| $\angle B =$ | ☐ degrees | b = | 11 |
| $\angle C =$ | ☐ degrees | c = | 14 |

$$\cos(\angle A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(\angle A) = \frac{(11)^2 + (14)^2 - (8)^2}{2(11)(14)}$$

$$\cos(\angle A) = \frac{121 + 196 - 64}{308}$$

$$\cos(\angle A) = \frac{253}{308}$$

$$m(\angle A) = \cos^{-1}\left(\frac{253}{308}\right) \approx 34.772^\circ$$

$$\cos(\angle B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(\angle C) = \frac{a^2 + b^2 - c^2}{2ab}$$