solve for θ:

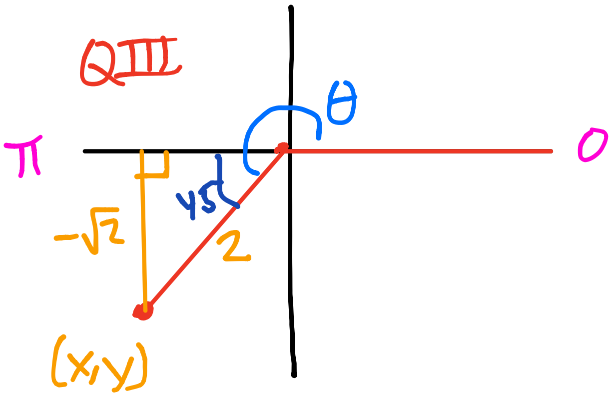
$$\sin\theta = -\frac{\sqrt{2}}{2}$$

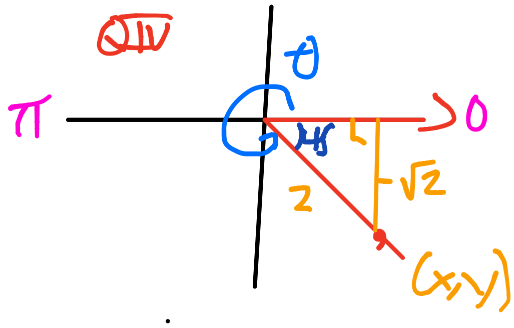
" $\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ "

Observation

$\sin\theta$ is negative

θ must be in QIII or QIV





$$\sin\theta_r = \frac{\sqrt{2}}{2}$$

$$m\angle\theta_r = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$m\angle\theta_r = 45^\circ = \frac{\pi}{4} \text{ radians}$$

note: not our angle

in QIII

$\theta = 180 + \theta_r$

$\theta = 180 + 45^\circ$

$\theta = 225^\circ$

you can convert from degrees to radians

in radians

$\theta = \pi + \theta_r$

$\theta = \pi + \frac{\pi}{4}$

$\theta = \frac{4\pi}{4} + \frac{\pi}{4}$

$\theta = \frac{5\pi}{4}$

in QIV

$\theta = 360 - \theta_r$

$= 360 - 45$

$= 315^\circ$

in radians

$\theta = 2\pi - \theta_r$

$\theta = 2\pi - \frac{\pi}{4}$

$\theta = \frac{8\pi}{4} - \frac{\pi}{4}$

$\theta = \frac{7\pi}{4}$

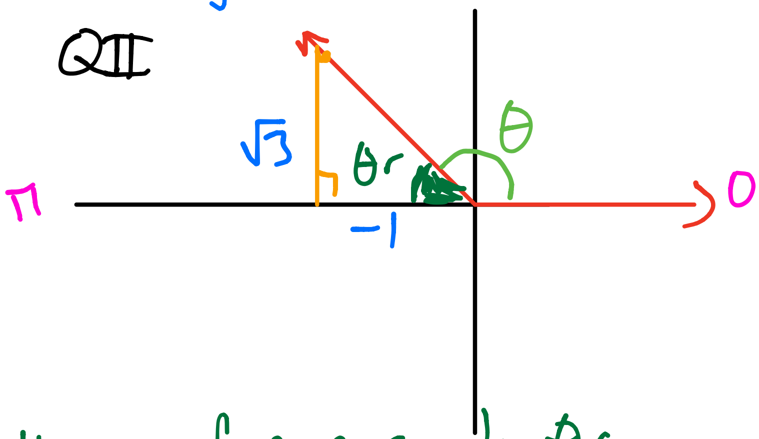
$$\boxed{m(\angle\theta) = \frac{5\pi}{4}, \frac{7\pi}{4}}$$

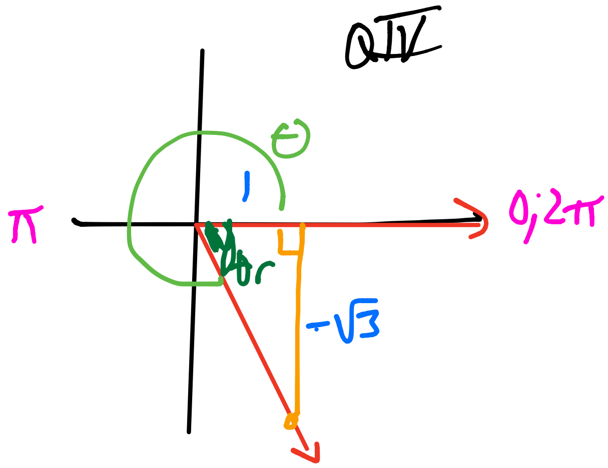
$\tan\theta = -\frac{\sqrt{3}}{1}$

"$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

$\tan\theta$ is negative

$\rightarrow$ QII , QIV





Using reference angle θr

$$\tan\theta_r = \frac{\sqrt{3}}{1}$$

$$\theta_r = \tan^{-1}(\sqrt{3})$$

$$\theta_r = 60^\circ = \frac{\pi}{3} \text{ radians}$$

QII

$\theta = 180^\circ - \theta r$

$= 180^\circ - 60^\circ$

$= 120^\circ$

convert to radians $\rightarrow$

in radians

$\theta = \pi - \theta r$

$\theta = \pi - \left(\frac{\pi}{3}\right)$

$\theta = \frac{3\pi}{3} - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

QIV

$\theta = 360 - \theta r$

$\theta = 2\pi - \theta r$

$\theta = 2\pi - \frac{\pi}{3}$

$\theta = \frac{6\pi}{3} - \frac{1\pi}{3}$

$\theta = \frac{5\pi}{3}$

$$\boxed{\theta = \frac{2\pi}{3}, \frac{5\pi}{3}}$$

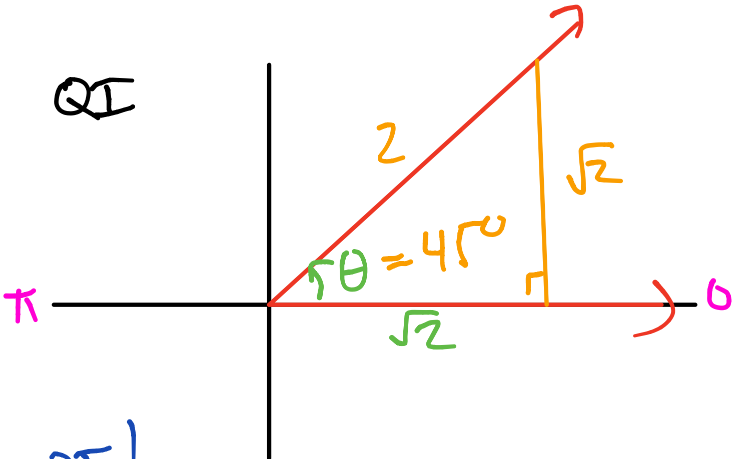
$$2\sin(x) - \sqrt{2} = 0$$
$$+\sqrt{2} \quad +\sqrt{2}$$

$$2\sin(x) = \sqrt{2}$$

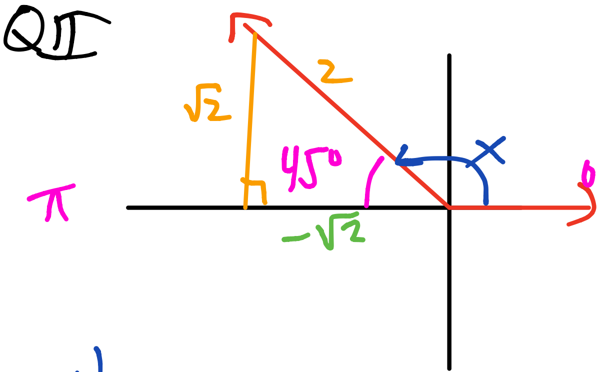
$$\sin(x) = +\frac{\sqrt{2}}{2}$$

$\rightarrow \sin(x)$ is positive QI, QII

QI



QII



QI

$$\sin(x) = \frac{\sqrt{2}}{2}$$

$$x = 45° = \frac{\pi}{4}$$

QII

$$x = \pi - \frac{\pi}{4}$$

$$x = \frac{3\pi}{4}$$

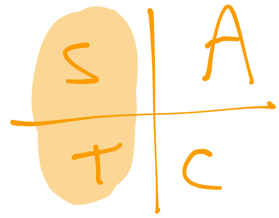
---

in QI, if $0 < \theta < 90$
$0 < \theta < \frac{\pi}{2}$

$\rightarrow \theta = \theta_r$

It's already acute, thus it is a reference angle also.

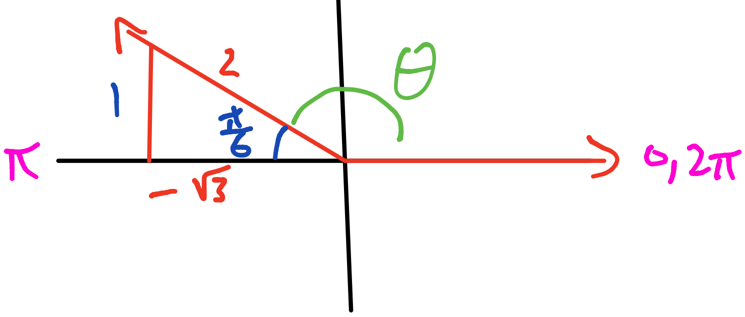
$$2\cos(x) + \sqrt{3} = 0$$
$$\quad -\sqrt{3} \quad -\sqrt{3}$$
$$2\cos(x) = -\sqrt{3}$$
$$\cos(x) = -\frac{\sqrt{3}}{2}$$



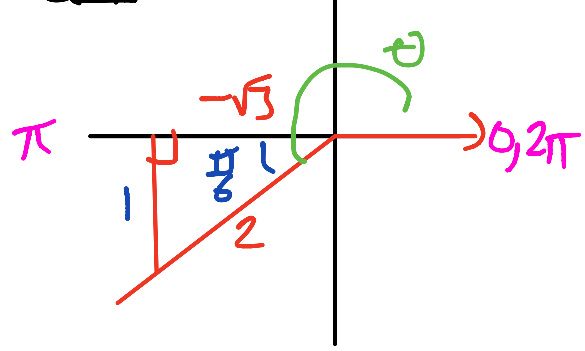
cos(x) is negative

→ QII, QIII

QII



QIII



$$\cos(x_r) = \frac{\sqrt{3}}{2}$$
$$x_r = \frac{\pi}{6}$$

← Find reference angle
$$x_r = \arccos\left(\frac{\sqrt{3}}{2}\right)$$

QII
$$x = \pi - x_r$$
$$x = \pi - \left(\frac{\pi}{6}\right)$$
$$x = \left(\frac{6\pi}{6}\right) - \frac{\pi}{6}$$
$$x = \frac{5\pi}{6}$$

QIII
$$x = \pi + x_r$$
$$= \pi + \left(\frac{\pi}{6}\right)$$
$$= \frac{6\pi}{6} + \frac{\pi}{6}$$
$$x = \frac{7\pi}{6}$$

$\tan^2\theta - 1 = 0$

Let $u = \tan\theta$

$u^2 - 1 = 0$

$$\begin{aligned} u^2 - 1 &= 0 \\ +1 \quad &+1 \\ \hline u^2 &= 1 \end{aligned}$$

$u = \pm 1$

$u^2 - 1 = 0$

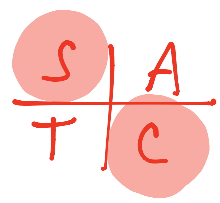
$(u+1)(u-1) = 0$

$$\begin{aligned} u+1 &= 0 \\ -1 \quad &-1 \\ \hline u &= -1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} u-1 &= 0 \\ +1 \quad &+1 \\ \hline u &= 1 \end{aligned}$$

Substitute $u = \tan\theta$

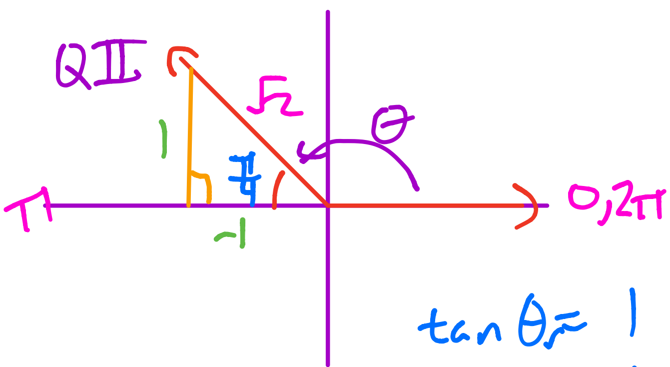
$u = -1$

$\tan\theta = -1$

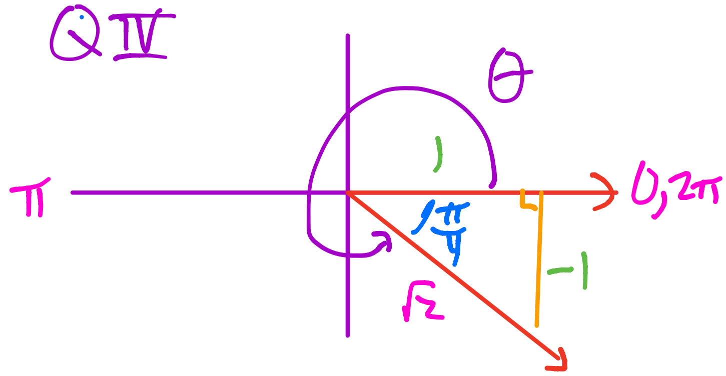


$\tan\theta$ is negative

↳ θ is in QII, QIV



QIV



$\tan\theta_r = 1$

$\theta_r = \tan^{-1}(1)$

$\theta_r = 45^\circ = \frac{\pi}{4}$

$\boxed{\text{QII}}$ $\theta = 180 - \theta_r$

$\theta = \pi - \theta_r$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\boxed{\text{QIV}}$ $\theta = 360 - \theta_r$

$\theta = 2\pi - \theta_r$

$\theta = 2\pi - \frac{\pi}{4}$

$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

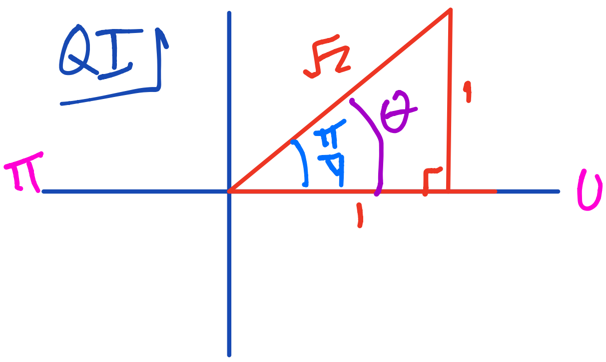
$u = 1$

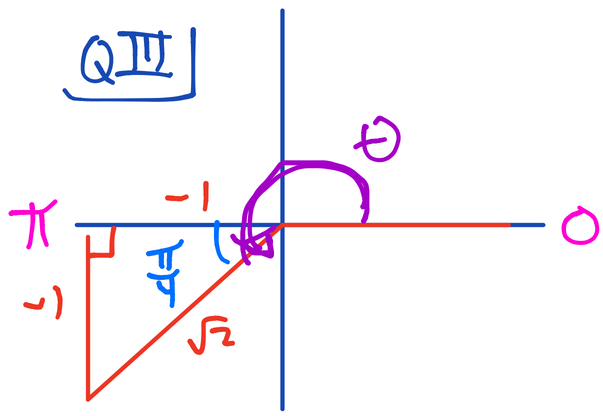
$\tan\theta = +1 \quad \rightarrow$ tan θ is positive in QI, QIII

$\tan\theta_r = 1$

$\theta_r = \tan^{-1}(1)$

$\theta_r = 45^0 = \frac{\pi}{4}$





QI $\theta = \theta r$

$\theta = \frac{\pi}{4}$

QIII $\theta = \pi + \theta r$

$\theta = \pi + \frac{\pi}{4}$

$\theta = \frac{4\pi}{4} + \frac{1\pi}{4}$

$\theta = \frac{5\pi}{4}$

$\theta \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

$\cos\theta \tan\theta - \tan\theta = 0$ $\quad 0 < \theta < 2\pi$

$(0, 2\pi)$

$\tan\theta (\cos\theta - 1) = 0$

$\tan\theta = 0$ or $\cos\theta - 1 = 0$

$\frac{\sin\theta}{\cos\theta} = \frac{0}{\cos\theta}$

$\sin\theta = 0$

$\theta = 0, 180, \cancel{360}$

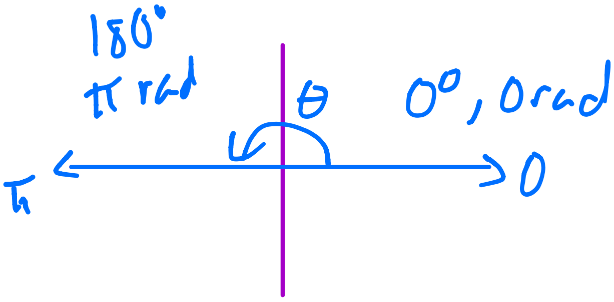
$= 0, \pi, \cancel{2\pi}$

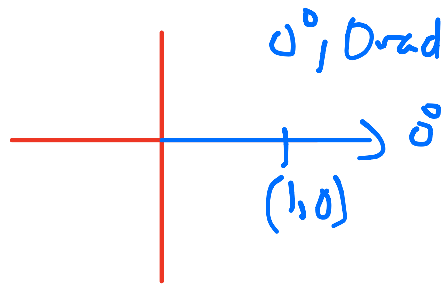
$\cos\theta - 1 = 0$
$\quad +1 \; +1$

$\cos\theta = 1$

$\theta = 0, \cancel{360}$

$\theta = 0, \pi$





$$2\sin^2(x) + 9\cos(x) + 3 = 0$$

$$2\left(1 - \cos^2(x)\right) + 9\cos(x) + 3 = 0$$

$$2 - 2\cos^2(x) + 9\cos(x) + 3 = 0$$

$$-2\cos^2(x) + 9\cos(x) + 5 = 0$$

$u = \cos(x)$

$$\frac{-2u^2 + 9u + 5 = 0}{-1 \qquad\qquad -1}$$

$$2u^2 - 9u - 5 = 0$$

$$(2u+1)(u-5) = 0$$

$$2\cos^2(x) - 9\cos(x) - 5 = 0$$

$$(2\cos(x) + 1)(\cos(x) - 5) = 0$$

Zero Product Property

$2\cos(x) + 1 = 0$ or $\cos(x) - 5 = 0$

$\cos(x) = 5$

* Note: $-1 \le \cos(x) \le 1$

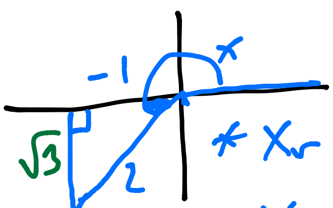
$\cos(x) \ne 5$

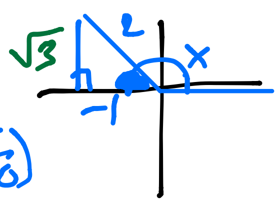
→ Reject

$2\cos(x) = -1$

$\cos(x) = -\frac{1}{2} = \text{"}\frac{x}{r}\text{"}$

* $\cos(x) < 0$ in QII, QIII





* $x_r = 60°\left(\frac{\pi}{180}\right)$

$x_r = \frac{\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$2\cos(x)\sin(x) - \sqrt{2}\sin(x) = 0$ over $[0, 2\pi)$

$(2\cos(x) - \sqrt{2})\sin(x) = 0$

$ab = 0$

$2\cos(x) - \sqrt{2} = 0$ or $\sin(x) = 0$

Let $u = \cos(x)$

$2u - \sqrt{2} = 0$

$2u = \sqrt{2}$

$u = \frac{\sqrt{2}}{2}$

$\cos(x) = \frac{\sqrt{2}}{2}$

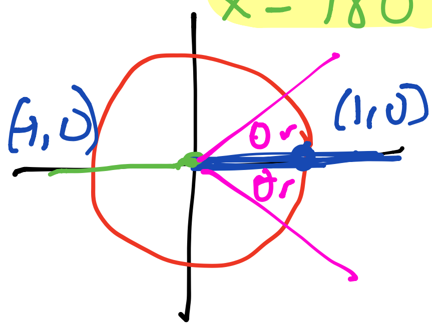
$\cos(x_r) = \frac{\sqrt{2}}{2}$

$x_r = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$x_r = 45^\circ$

$x = 0^\circ \left(\frac{\pi}{180}\right) = 0\text{ rad}$

$x = 180^\circ \left(\frac{\pi}{180}\right) = \pi\text{ rad}$



cos(x) is positive in

QI – (0, 90)

QIV – (270, 360)

QI solution

$x = x_r$

$x = 45^\circ \left(\frac{\pi}{180}\right) = \frac{\pi}{4}$

QIV solution

$x = 360^\circ - 45^\circ$

$x = 315^\circ \left(\frac{\pi}{180}\right) = \frac{7\pi}{4}$

$x \in \{0^\circ, 45^\circ, 180^\circ, 315^\circ\}$

$x \in \left\{0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}\right\}$