

Recall.

$$y = ax^2 + bx + c \leftarrow \text{standard form}$$

$$y = a(x - h)^2 + k \leftarrow \text{vertex form}$$

(h, k) is vertex

Converting via completing the square

$$y = ax^2 + bx + c$$

$$y - c = ax^2 + bx$$

$$\frac{y - c}{a} = x^2 + \frac{b}{a}x$$

$$\frac{y - c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$$

$$\frac{y - c}{a} + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{y - c}{a} = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$

$$y - c = a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} - c$$

$$y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(\frac{4ac - b^2}{4a}\right)$$