Nonlinear Systems of Equations

At least one equation is not a line

Methods of solving Systems of Linear Equations

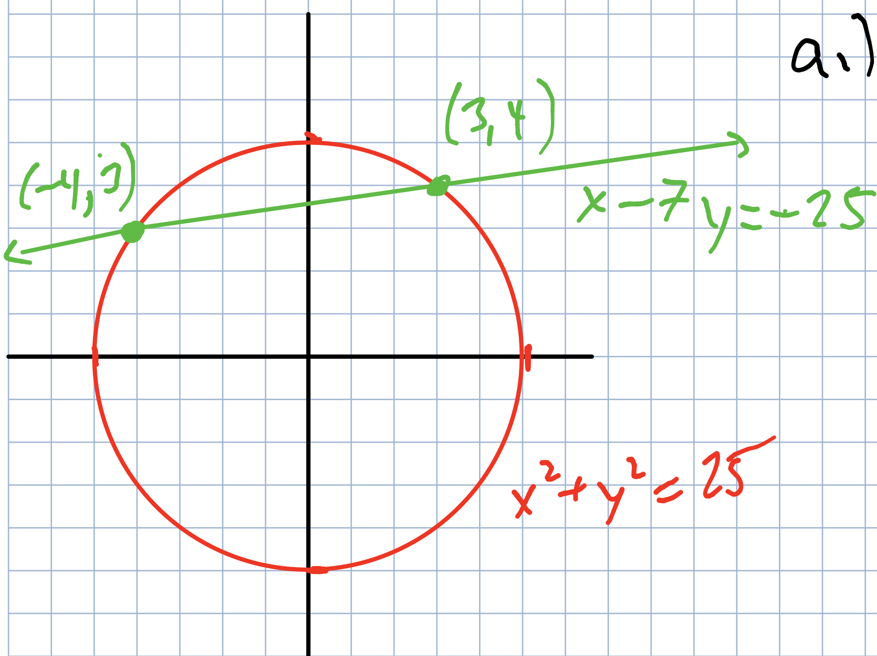
Substitution

Elimination / Addition Method

$x - 7y = -25$ ← line in $Ax + By = C$ form

$x^2 + y^2 = 25$ ← circle radius: 5
center: $(0,0)$

a.) by graphing



$$x - 7y = -25$$
$$\underline{-x \qquad\quad -x}$$
$$-7y = -x - 25$$
$$y = \frac{1}{7}x + \frac{25}{7}$$

(Note: not easy to graph)

Our solutions: $(-4,3)$, $(3,4)$

$$x - 7y = -25$$
$$x^2 + y^2 = 25$$

Substitution Method

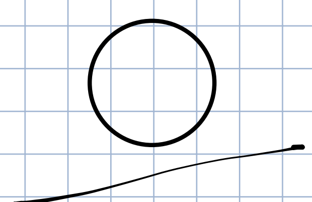
Solve for x.

$$x - 7y = -25$$
$$x = 7y - 25$$

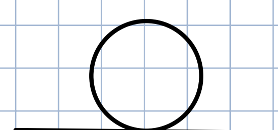
$$\boxed{\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2\end{aligned}}$$

$$(7y-25)^2 + y^2 = 25$$
$$\downarrow$$
$$(7y)^2 - 2(25)(7y) + (-25)^2 + y^2 = 25$$
$$49y^2 - 350y + 625 + y^2 = 25$$
$$50y^2 - 350y + 625 = 25$$
$$-25 \qquad -25$$
$$50y^2 - 350y + 600 = 0$$
$$50(y^2 - 7y + 12) = 0$$
$$50(y-3)(y-4) = 0$$

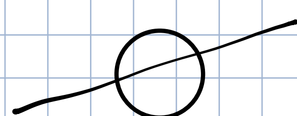
$$50 \neq 0 \quad \Big| \quad y - 3 = 0 \quad \text{or} \quad y - 4 = 0$$
$$y = 3 \qquad\qquad y = 4$$



0 solutions



1 solution



2 solutions

$x - 7y = -25$

Let $y = 3$

$$x - 7(3) = -25$$
$$x - 21 = -25$$
$$+21 \quad +21$$
$$x = -4$$

Let $y = 4$

$$x - 7(4) = -25$$
$$x - 28 = -25$$
$$+28 \quad +28$$
$$x = 3$$

$$(x, y) = (-4, 3), (3, 4)$$

$x^2 + y^2 = 25 \leftarrow$ avoid substituting here unless no other choice

Let $y = 3$

$$x^2 + (3)^2 = 25$$
$$x^2 + 9 = 25$$
$$-9 \quad -9$$
$$x^2 = 16$$
$$x = \pm 4$$

Let $y = 4$

$$x^2 + (4)^2 = 25$$
$$x^2 + 16 = 25$$
$$-16 \quad -16$$
$$x^2 = 9$$
$$x = \pm 3$$

$$(x, y) = (-4, 3), (4, 3), (-3, 4), (3, 4)$$

if we substitute back into circle,

we get extraneous solutions.

$x - 7y = -25$ $(x,y) = (4,3)$

$(4) - 7(3) = -25$

$4 - 21 = -21$

$-17 \neq -21$ ← did not check

→ (4, 3) is not an actual solution

*Try to avoid substituting back into circle, ellipse or hyperbola equation. unless you have no choice

$y=\sqrt{x}$ ← radical

$x^2+y^2=20$ ← circle center $(0,0)$ radius: $\sqrt{20}$

: $\cancel{\sqrt{4}}\sqrt{5}$

: $2\sqrt{5}$

↘

$x^2+(\sqrt{x})^2=20$

$x^2+x=20$

$x^2+x-20=0$

$(x-4)(x+5)=0$

$x-4=0$ or $x+5=0$

$x=4$ $\quad$ $x=-5$

Use $y=\sqrt{x}$

Let $x=4$ $\quad y=\sqrt{4}$

$y=2$

$(x,y)=(4,2)$

Let $x=-5$ $\quad y=\sqrt{-5}$

$y=\sqrt{5}i$

$(x,y)=(-5,\sqrt{5}i)$

*not a real solution

Cannot graph $\sqrt{5}i$

$y^2=x$

$y=\pm\sqrt{x}$

Not the same

$y = \sqrt[3]{x}$ $\longrightarrow$ $\sqrt[3]{x} = x$

$y = x$

$$\left(\sqrt[3]{x}\right)^3 = x^3$$

$$x = x^3$$

$$\begin{array}{rcl} x & = & x^3 \\ -x & & -x \\ \hline 0 & = & x^3 - x \end{array}$$

$$0 = x(x^2 - 1)$$

$$0 = x(x+1)(x-1)$$

$x = 0$	$x+1=0$	$x-1=0$
	$x = -1$	$x = 1$
$y = x$	
$y = 0$	$y = -1$	$y = 1$
$(0,0)$	$(-1,-1)$	$(1,1)$

$y = \sqrt[3]{x}$	$y = \sqrt[3]{-1}$	$y = \sqrt[3]{1}$
$y = \sqrt[3]{0}$	$y = -1$	$y = 1$
$y = 0$		
$(0,0)$	$(-1,-1)$	$(1,1)$

$$2x^2 + y^2 = 17 \leftarrow \text{ellipse}$$

$$x^2 + 2y^2 = 22$$

Elimination

$u = x^2$

$v = y^2$

A $2u + v = 17$

B $u + 2v = 22$

Choose to eliminate u

A $2u + v = 17$

-2B $-2u - 4v = -44$

$$-3v = -27$$

$$\frac{-3v}{-3} = \frac{-27}{-3}$$

$$v = 9$$

$y^2 = 9$

$y = \pm 3$

Solve for u

B $u + 2v = 22$

$u + 2(9) = 22$

$u + 18 = 22$

$u = 4$

$x^2 = 4$

$x = \pm 2$

We don't know how to connect (x, y)

Ⓐ $2x^2 + y^2 = 17$

Ⓑ $x^2 + 2y^2 = 22$

Ⓐ $2x^2 + y^2 = 17$ Choose to eliminate y

-2Ⓑ $-2x^2 - 4y^2 = -44$

$-3y^2 = -27$

$\frac{-3y^2}{-3} = \frac{-27}{-3}$

$y^2 = 9$

$y = \pm 3$

Solve for x

$x^2 + 2y^2 = 22$

Let $y = -3$

$x^2 + 2(-3)^2 = 22$

$x^2 + 2(9) = 22$

$x^2 + 18 = 22$

$-18 \quad -18$

$x^2 = 4$

$x = \pm 2$

Let $y = 3$

$x^2 + 2(3)^2 = 22$

$x^2 + 2(9) = 22$

$x^2 + 18 = 22$

$-18 \quad -18$

$x^2 = 4$

$x = \pm 2$

$(x,y) = (-2,-3), (2,-3), (-2,3), (2,3)$