$\sqrt{25}$ is 5 or -5, b/c $5^2 = 25$

$(-5)^2 = 25$

$\sqrt{49}$ is 7 or -7

$\sqrt{0} = 0$ only one $\sqrt{0}$ because 0 is neither positive or negative

$\sqrt{-9} \longrightarrow$ no $\sqrt{-9}$ b/c can't take square root of a negative number

* $\sqrt{a}$, if $a > 0$, then there are two square roots

$a = 0$, only 0 is square root

$a < 0$, no real square root

$\sqrt{a}$ * positive square root (principal root)

$-\sqrt{a}$ * negative square root

$\sqrt{36} = 6$ b/c $(6)^2 = 36$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$ $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ $\frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

Quotient property for radicals

$\sqrt{.04} = \sqrt{\frac{4}{100}} = \frac{2}{10} = \boxed{\frac{1}{5}} = \boxed{.2}$

Definition of n-th root –

b is an n-th root of a

if $b^n = a$

e.g., 2 is a square root of 4. $2^2 = 4$

2 is a cube root of 8 $2^3 = 8$

2 is a fourth root of 16 $2^4 = 16$

$\sqrt[n]{a}$ n – index → $n \in \{2, 3, 4, 5, \ldots\}$

a – radicand

e.g. $\sqrt{4} = 2$ $\sqrt[3]{8} = 2$ $\sqrt[4]{16} = 2$

no index, assume square root always

$\sqrt[3]{27} = 3$ b/c $3^3 = 27$

$\sqrt[3]{-27} = -3$ b/c $(-3)^3 = 27$

$\sqrt[4]{16} = 2$

$\sqrt[6]{64} = 2$

$\sqrt[4]{-16}$ = not real

$\sqrt[6]{-64}$ = not real

$\sqrt[5]{32} = 2$

$\sqrt[5]{-32} = -2$

* if index is even, radicand cannot be negative

* if index is odd, radicand can be negative

- if index is even, then $\sqrt[n]{a}$ is always the principal n-th root of a
- if index is odd, then $\sqrt[n]{a}$ is the n-th root of a,

$= \sqrt[n]{0} = 0$

Evaluate $\sqrt[n]{a^n}$

if n is odd, $\sqrt[n]{a^n} = a$

n is even, $\sqrt[n]{a^n} = |a|$

no negative sign

$\sqrt[4]{(-3)^4} = |-3| = 3$

even index

$\sqrt[5]{(-3)^5} = -3$

odd index

note

$x+2 \neq |x+2|$

$\sqrt[2]{(x+2)^2} = |x+2|$

even index

$\sqrt[3]{(a+b)^3} = a+b$

odd index

$\sqrt[2]{y^4} = \sqrt[2]{(y^2)^2} = |y^2| = y^2$

$y^4 = (y^2)^2$

even index

b/c y^2 can never be negative

$y^{\text{even number}}$ can not be negative

$\sqrt{y^8} = \sqrt[2]{(y^4)^2} = |y^4| = y^4$

$$\sqrt{y^6} = \sqrt[2]{(y^3)^2} = |y^3| \neq y^3$$

* keep the absolute value

a. even index

b. y^3 can be negative

*Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

same as multiplication property of exponents.

e.g. $\sqrt{36} = \sqrt{9 \cdot 4}$

$= \sqrt{9} \cdot \sqrt{4}$

$= 3 \cdot 2 = 6$

$\sqrt{36} = \sqrt{6 \cdot 6}$

$= \sqrt{6} \cdot \sqrt{6}$

$= (\sqrt{6})^2 = 6$

Simplified form of a radical

1. Radicand has no factor raised to a power greater than or equal to the index.

2. Radicand doesn't contain a fraction

3. No radicals in denominator of a fraction

$\sqrt{x^2} = |x|$ ← "actual answer"

$\sqrt{x^6} = |x^3|$

$\sqrt{x^4} = x^2$

$\sqrt{x^8} = x^4$

$\sqrt{x^2} = x$ only when assuming $x > 0$

x is positive

$\sqrt{x^6} = x^3$

What about odd power radicands?

$\sqrt{x^9} = \sqrt[2]{x^8}\ \sqrt[2]{x^1}$ ← factor's power is less than index

factor's power is largest power divisible by index

$= x^4 \sqrt{x}$

$\sqrt{a^{11}} = \sqrt[2]{a^{10}}\ \sqrt{a}$

$= |a^5| \sqrt{a}$

$\sqrt{a^{11}} = \sqrt{a^{10}}\sqrt{a}$ assume $a > 0$

$= a^5 \sqrt{a}$

$|a^5| \neq a^5$

$\sqrt[4]{r^{27}} = \sqrt[4]{r^{24}} \cdot \sqrt[4]{r^3}$

$= \sqrt[4]{(r^6)^4} \cdot \sqrt[4]{r^3}$

$= r^6 \sqrt[4]{r^3}$

$* r^{24} \cdot r^2 = r^{24+2}$

$= r^{26}$

$|r^6| = r^6$

*Assume all variables are positive

(2) $\sqrt{w^7 \cdot z^9} = \sqrt{w^7}\ \sqrt{z^9}$

$= \sqrt{w^6}\ \sqrt{w^1}\ \sqrt{z^8}\ \sqrt{z^1}$

$= \sqrt{w^6}\ \sqrt{z^8}\ \sqrt[2]{w^1}\sqrt[2]{z^1}$

$= w^3 \cdot z^4 \sqrt{wz}$

$\sqrt{40x^{17}y^{10}} = \sqrt{2^3 \cdot 5\ x^{17}\ y^{10}}$

$= \sqrt[2]{2^2 x^{16} y^{10}}\ \sqrt[2]{2^1 \cdot 5^1 x^1}$

$= \sqrt[2]{2^2 (x^8)^2 (y^5)^2}\ \sqrt{2 \cdot 5x}$

$= 2x^8y^5\sqrt{10x}$

40

4 10

2 2 5 2

$= 2^3 \cdot 5$

$$\sqrt{\frac{a^7}{a^3}} = \sqrt{a^4} = a^2$$

$$\frac{7\sqrt{50}}{10} = \frac{7}{10}\sqrt{25}\cdot\sqrt{2}$$

$$= \frac{7}{10}\cdot 5\sqrt{2}$$

$$= \frac{35\sqrt{2}}{10}$$

$$= \frac{7}{2}\sqrt{2}$$

*Can we multiply 7 by $\sqrt{2}$?

$7\sqrt{2} = \sqrt{14}$?

No

because different powers / roots

Rational Exponents

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

eg $4^{\frac{1}{2}} = \sqrt{4}$

$6^{\frac{1}{3}} = \sqrt[3]{6}$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

eg. $4^{\frac{3}{2}} = \sqrt{4^3}$ $= \left(\sqrt{4}\right)^3$

$= \sqrt{64}$ $= 2^3$

$= 8$ $= 8$

$81^{\frac{3}{4}} = \sqrt[4]{81^3}$ $= \left(\sqrt[4]{81}\right)^3$

$= \sqrt[4]{(3^4)^3}$ $= \left(\sqrt[4]{3^4}\right)^3$

$= \sqrt[4]{(3^3)^4}$ $= 3^3$

$= 3^3 = 27$ $= 27$