Prove:

$$1+\cos(A) = \frac{\sin^2(A)}{1-\cos(A)}$$

RHS

$$\frac{\sin^2(A)}{1-\cos(A)} \cdot \frac{1+\cos(A)}{1+\cos(A)}$$

[Multiply by Conjugate].

$$\frac{\sin^2(A)\left(1+\cos(A)\right)}{1-\cos^2(A)}$$

$[(a-b)(a+b)=a^2-b^2]$

$$\frac{\sin^2(A)\left(1+\cos(A)\right)}{\left(\sin^2(A)+\cos^2(A)\right)-\cos^2(A)}$$

$[1=\sin^2(A)+\cos^2(A)]$

$$\frac{\sin^2(A)\left(1+\cos(A)\right)}{\sin^2(A)}$$

$$1+\cos(A) \quad \square$$

Prove the identity $\tan(B) + \cot(B) = \sec(B)\csc(B)$

LHS $\quad \tan(B) + \cot(B)$

$$\frac{\sin(B)}{\cos(B)} + \frac{\cos(B)}{\sin(B)}$$

$$\frac{\sin(B)}{\sin(B)}\,\frac{\sin(B)}{\cos(B)} + \frac{\cos(B)}{\sin(B)}\,\frac{\cos(B)}{\cos(B)} \qquad [\text{LCD: } \sin(B)\cos(B)]$$

$$\frac{\sin^2(B) + \cos^2(B)}{\sin(B)\cos(B)}$$

$$\frac{1}{\sin(B)\cos(B)}$$

$$\frac{1}{\sin(B)} \cdot \frac{1}{\cos(B)}$$

$$\csc(B) \quad \sec(B)$$

$$\sec(B) \cdot \csc(B)$$

QED

Prove: $\sin(x)\cdot\cot^2(x)+\sin(x)=\csc(x)$

LHS $\quad \sin(x)\cdot\cot^2(x)+\sin(x)$

$\sin(x)\left(\cot^2(x)+1\right)$

$\sin(x)\ \csc^2(x)$ $\quad$ $[\cot^2(x)+1=\csc^2(x)]$

$\sin(x)\ \csc(x)\cdot\csc(x)$ $\quad$ $[a^2=a\cdot a]$

$\sin(x)\cdot\dfrac{1}{\sin(x)}\cdot\csc(x)$ $\quad$ $\left[\csc(x)=\dfrac{1}{\sin(x)}\right]$

$\csc(x)$ $\square$

Prove: $\sin^2(a) = \dfrac{1-\cos^4(a)}{1+\cos^2(a)}$

RHS $\dfrac{1-\cos^4(a)}{1+\cos^2(a)}$ [difference of squares]

$$\frac{\cancel{(1+\cos^2(a))}\,(1-\cos^2(a))}{\cancel{1+\cos^2(a)}}$$

$$1-\cos^2(a)$$

$$\sin^2(a)+\cos^2(a)-\cos^2(a)$$

$$\sin^2(a)$$ QED

Prove: $\dfrac{1+\tan(y)}{1+\cot(y)} = \tan(y)$

LHS $\dfrac{1+\tan(y)}{1+\cot(y)}$

$$\dfrac{1+\dfrac{\sin(y)}{\cos(y)}}{1+\dfrac{\cos(y)}{\sin(y)}}$$

$$\left(\dfrac{\sin(y)\cos(y)}{\sin(y)\cos(y)}\right)\left(\dfrac{1+\dfrac{\sin(y)}{\cos(y)}}{1+\dfrac{\cos(y)}{\sin(y)}}\right)$$

[Complex Fraction
LCD: $\sin(y)\cos(y)$]

$$\dfrac{\sin(y)\cos(y)+\sin^2(y)}{\sin(y)\cos(y)+\cos^2(y)}$$

$$\dfrac{\sin(y)\,(\cos(y)+\sin(y))}{\cos(y)\,(\sin(y)+\cos(y))}$$

$$\tan(y)$$

QED.

Prove: $\frac{1}{\cos(B)} - \cos(B) = \sin(B)\tan(B)$

LHS $\frac{1}{\cos(B)} - \cos(B)$ [LCD: $\cos(B)$]

$$\frac{1}{\cos(B)} - \frac{\cos(B)\cos(B)}{\cos(B)}$$

$$\frac{1-\cos^2(B)}{\cos(B)}$$ [$1-\cos^2(B)=\sin^2(B)$]

$$\frac{\sin^2(B)}{\cos(B)}$$

$$\frac{\sin(B)}{1}\cdot\frac{\sin(B)}{\cos(B)}$$

$\sin(B)\tan(B)$ QED.

Prove: $$2\csc^2(t) = \frac{1}{1-\cos(t)} + \frac{1}{1+\cos(t)}$$

RHS $$\frac{1}{1-\cos(t)} + \frac{1}{1+\cos(t)}$$

$$\frac{(1+\cos(t))}{(1+\cos(t))}\left(\frac{1}{1-\cos(t)}\right) + \left(\frac{1}{1+\cos(t)}\right)\cdot\frac{(1-\cos(t))}{(1-\cos(t))}$$

$$\frac{1+\cos(t) + 1-\cos(t)}{(1+\cos(t))\,(1-\cos(t))}$$

$$\frac{2}{1-\cos^2(t)}$$

$$\frac{2}{\sin^2(t)}$$

$$2\cdot\left(\frac{1}{\sin^2(t)}\right)$$

$$2\cdot\csc^2(t) \quad \square$$