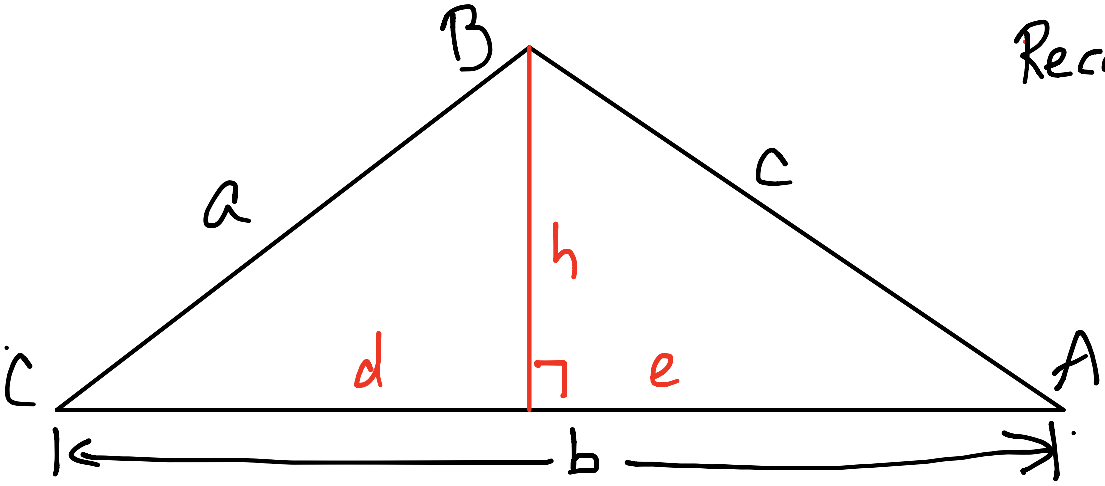


Recall: $b = d + e$

$\rightarrow e = b - d$

$d = b - e$

$$\cos(\angle C) = \frac{d}{a}$$

$$d = a\cos(\angle C)$$

$$\sin(\angle C) = \frac{h}{a}$$

$$h = a\sin(\angle C)$$

By the Pythagorean Theorem,

$$c^2 = h^2 + e^2$$

$$c^2 = h^2 + (b-d)^2$$

$$c^2 = (a\sin(\angle C))^2 + (b - a\cos(\angle C))^2$$

$$c^2 = a^2\sin^2(\angle C) + (b^2 + a^2\cos^2(\angle C) - 2ab\cos(\angle C))$$

$$c^2 = a^2\sin^2(\angle C) + a^2\cos^2(\angle C) + b^2 - 2ab\cos(\angle C)$$

$$c^2 = a^2(\sin^2(\angle C) + \cos^2(\angle C)) + b^2 - 2ab\cos(\angle C)$$

$$c^2 = a^2(1) + b^2 - 2ab\cos(\angle C)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$\rightarrow a^2 = b^2 + c^2 - 2bc\cos(\angle A)$ (by similar logic)

$\rightarrow b^2 = a^2 + b^2 - 2bc\cos(\angle B)$

* The Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$
$$-(a^2+b^2) \quad -(a^2+b^2)$$

$$c^2 - (a^2+b^2) = -2ab\cos(\angle C)$$

$$\frac{c^2 - (a^2+b^2)}{-2ab} = \frac{-2ab\cos(\angle C)}{-2ab}$$

$$\frac{c^2 - (a^2+b^2)}{-2ab} = \cos(\angle C)$$

$$\frac{a^2+b^2-c^2}{2ab} = \cos(\angle C)$$

$$\rightarrow \frac{b^2+c^2-a^2}{2bc} = \cos(\angle A)$$

$$\rightarrow \frac{a^2+c^2-b^2}{2ac} = \cos(\angle B)$$

SSS

3 sides find angle

SAS

2 sides
angle in between
Find Last side.

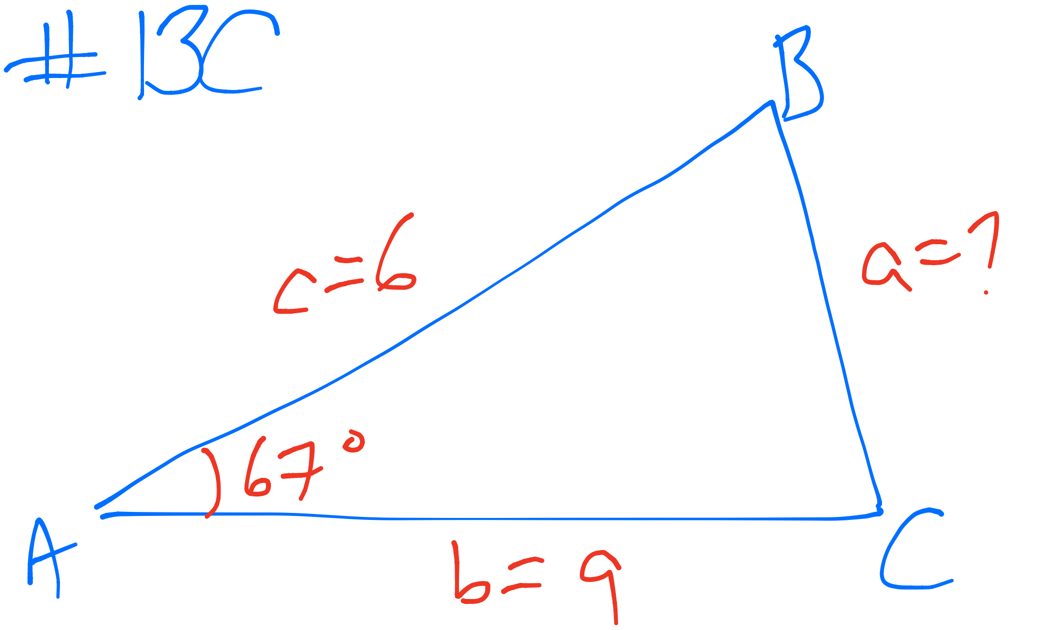
FE Review #13C

$b=9$

$c=6$

$\angle A = 67^\circ$

Find side A



SAS → Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\angle A)$$

$$a^2 = (9)^2 + (6)^2 - 2(9)(6)\cos(67)$$

$$a^2 = 117 - 108\cos(67)$$

$$a^2 = \sqrt{117 - 108\cos(67)}$$

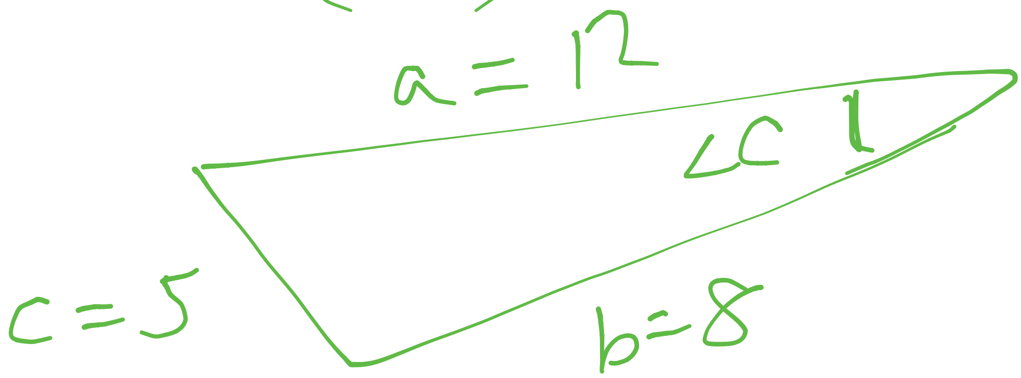
$$a \approx 8.649$$

FE Review #13A

$a = 12$ Find$(\angle C)$

$b = 8$

$c = 5$



$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$$(5)^2 = (12)^2 + (8)^2 - 2(12)(8)\cos(\angle C)$$

$$25 = 208 - 192\cos(\angle C)$$

$$-208 \qquad -208$$

$$\frac{-183}{-192} = \frac{-\cancel{192}\cos(\angle C)}{-\cancel{192}}$$

$$\cos(\angle C) = \frac{183}{192}$$

$$m\angle C = \cos^{-1}\left(\frac{183}{192}\right)$$

$$\boxed{m\angle C \approx 17.612^\circ}$$