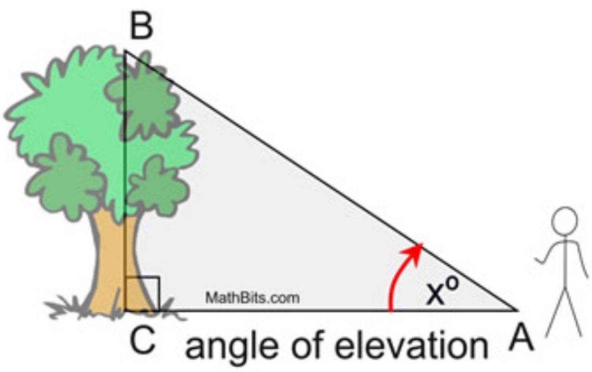
Angle of Elevation:



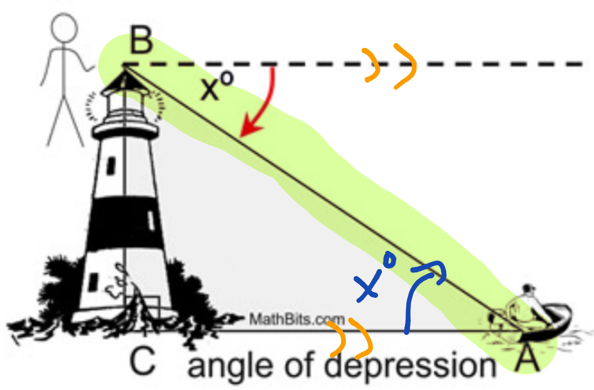
In this diagram, x° marks the angle of elevation of the top of the tree as seen from a point on the ground.

The angle of elevation is always measured from the *ground up*. It is an upward angle from a horizontal line. It is always **inside** the triangle.

You can think of the angle of elevation in relation to the movement of your eyes. You are looking straight ahead and you must raise (*elevate*) your eyes to see the top of a tree.

When trying to remember the meaning of an angle of elevation think of an *elevator* that only goes up!

Angle of Depression:



In this diagram, x° marks the angle of depression of the boat at sea from the top of the lighthouse.

The angle of depression is always **OUTSIDE** the triangle. It is never inside the triangle. It is a downward angle from a horizontal line.

You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse and you are looking straight ahead. You must lower (*depress*) your eyes to see the boat in the water.

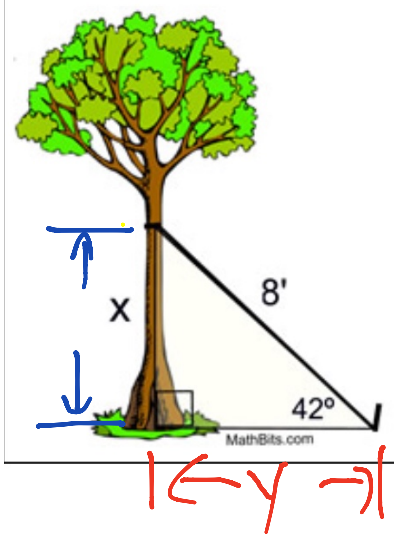
angle of elevation = angle of depression

by Alternate Interior Angles of Parallel lines

Notice how the horizontal line in the angle of depression diagram is PARALLEL to the ground level. The fact that horizontal lines are always parallel guarantees that the alternate interior angles are equal in measure. In the diagram, the angle marked x° is equal in measure to $m\angle BAC$. Simply stated, this means that ...

→ the angle of elevation = the angle of depression ←

A nursery plants a new tree and attaches a guy wire to help support the tree while its roots take hold. An eight foot wire is attached to the tree and to a stake in the ground. From the stake in the ground the angle of elevation of the connection with the tree is 42º. Find to the *nearest tenth of a foot*, the height of the connection point on the tree.



Given 42° angle of elevation

opposite : x

hypotenuse : 8ft

$$\sin\theta = \frac{opp}{hyp}$$

$$\frac{\sin(42^\circ)}{1} = \frac{x}{8\text{ft}}$$

$$\boxed{8\sin(42^\circ)\text{ ft} = x}$$

*Make sure calculator is in degree mode

$$x \approx 5.35\text{ft}$$

$$\boxed{\rightarrow x \approx 5.4\text{ft}}$$

Find y: Use Pythagorean Theorem

$$x^2 + y^2 = r^2$$

$$(8\sin(42))^2 + y^2 = 8^2$$

$$64\sin^2(42) + y^2 = 64$$

$$y^2 = 64 - 64\sin^2(42)$$

$$y = \sqrt{64 - 64\sin^2(42)}$$

still more unrevealed stuff

y = adjacent

8ft hypotenuse

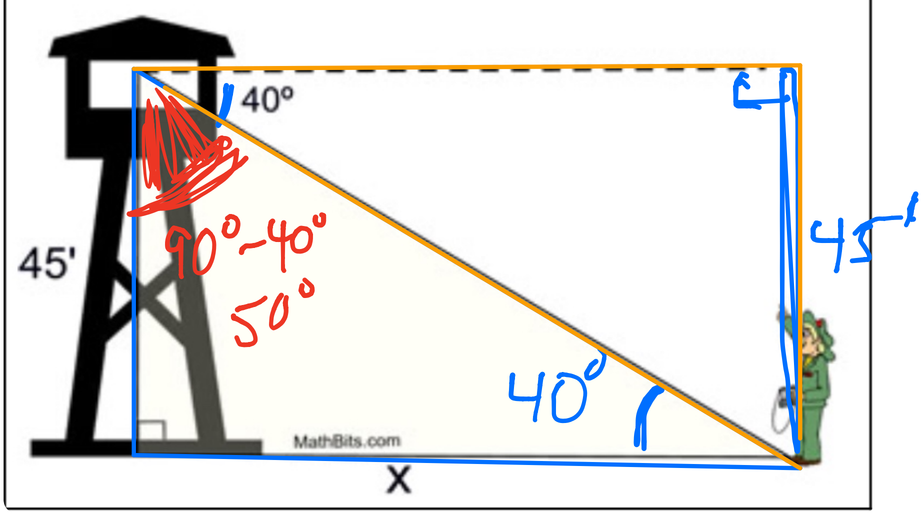
$$\cos(42) = \frac{adj}{hyp}$$

$$\frac{\cos(42)}{1} = \frac{y}{8\text{ft}}$$

$$y = 8\cos(42)$$

$$y \approx 5.94\text{ft}$$

From the top of a fire tower, a forest ranger sees his partner on the ground at an angle of depression of 40º. If the tower is 45 feet in height, how far is the partner from the base of the tower, to the *nearest tenth of a foot*?



$45' = adj \quad \theta = 50°$

$x = opp$

$\tan\theta = \frac{opp}{adj}$

$\frac{\tan(50)}{1} = \frac{x}{45}$

$x = 45\tan(50)$

$\boxed{x \approx 53.6 ft}$

$45' = opp$

$x = adj$

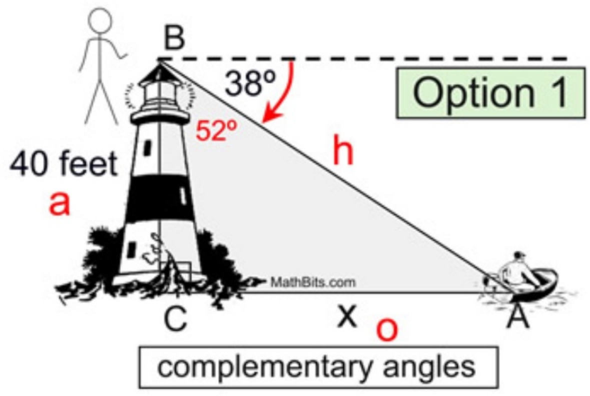
$\tan\theta = \frac{opp}{adj}$

$\frac{\tan(40)}{1} = \frac{45}{x}$

$\frac{x\cancel{\tan(40)}}{\cancel{\tan(40)}} = \frac{45}{\tan(40)}$

$x = \frac{45}{\tan(40)}$

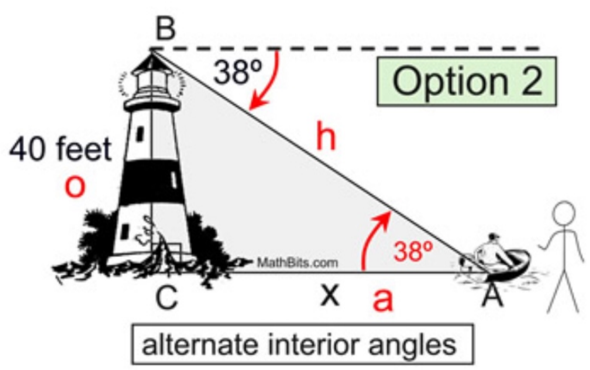
$x \approx 53.6 ft$



When solving a problem with an angle of depression you need to find the measure of an angle **INSIDE** the triangle. There are two options:

Option 1: find the angle **inside** the triangle that is adjacent (next door) to the angle of depression. This adjacent angle will always be the complement of the angle of depression, since the horizontal line and the vertical line are perpendicular (90º). In the diagram at the left, the adjacent angle is 52º.

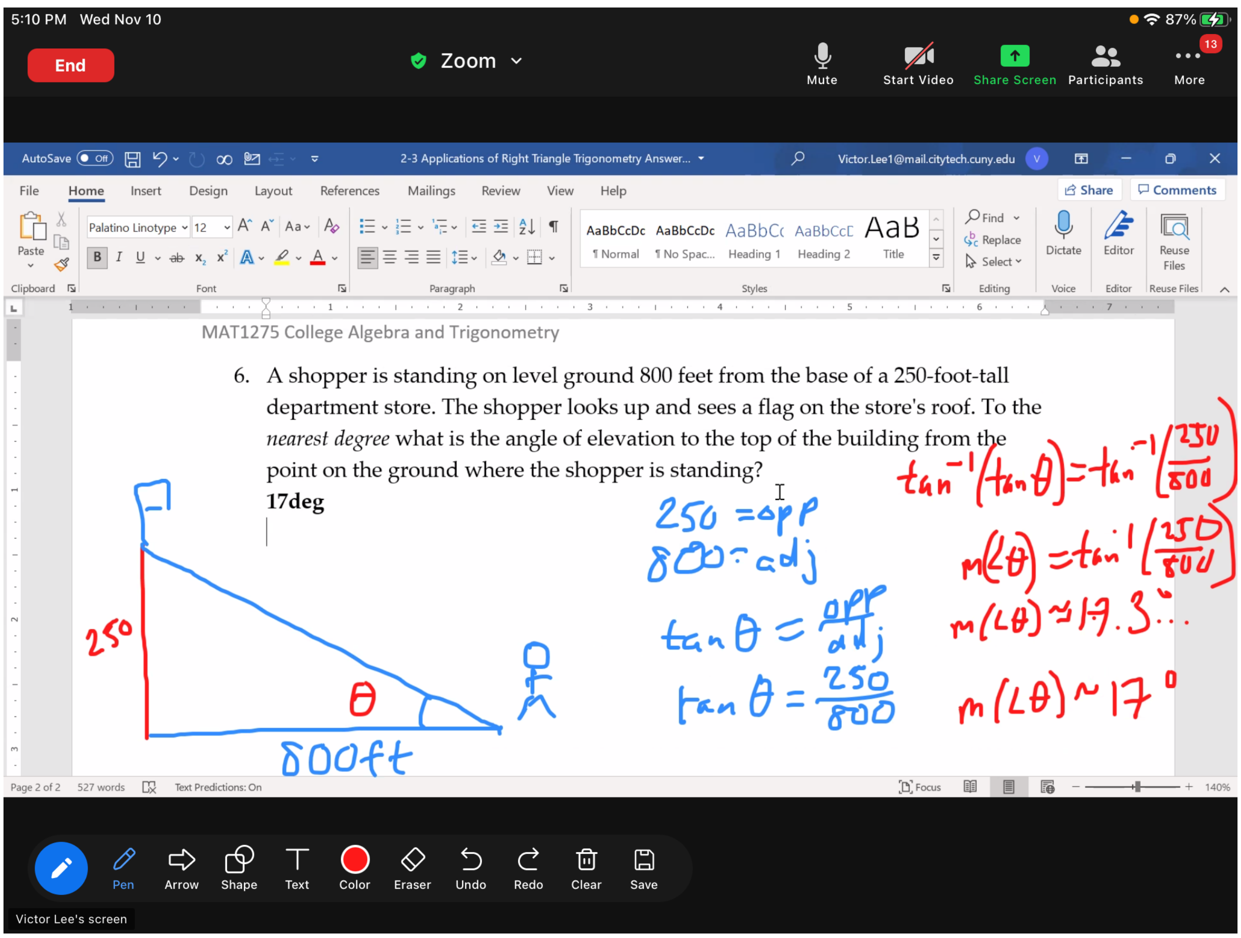
$$\tan 52^\circ = \frac{opposite}{adjacent} = \frac{x}{40}; \quad 1.279941632 = \frac{x}{40}; \quad x \approx 51 \text{ ft.}$$



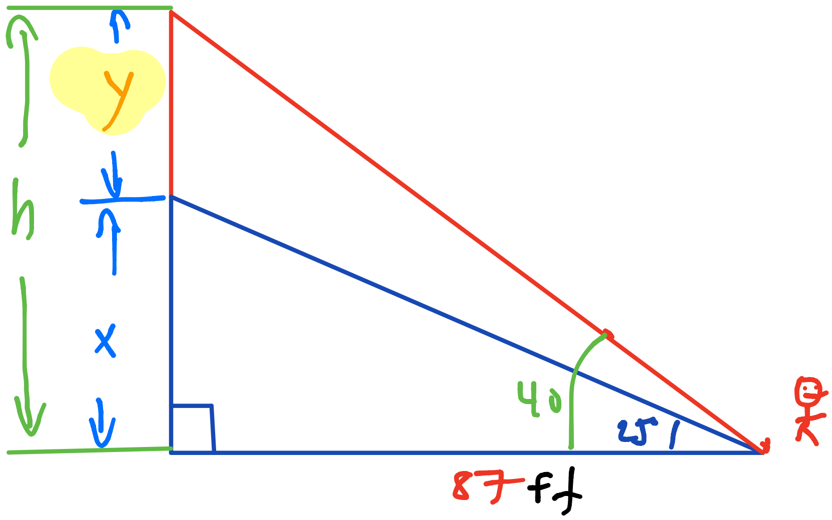
Option 2: utilize the fact that the angle of depression = the angle of elevation and label $\angle BAC$ as 38º **inside** the triangle.

$$\tan 38^\circ = \frac{opposite}{adjacent} = \frac{40}{x}; \quad 0.7812856265 = \frac{40}{x}; \quad x \approx 51 \text{ ft.}$$

Notice that both options, the answer is the same.



A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25º, and the angle of elevation of the top of the second section is 40º. To the *nearest foot*, what is the height of the top section of the tower?



Need to solve for y

1. Solve for x.

$\tan\theta = \frac{opp}{adj}$

opp: x
adj = 87 ft
$\theta = 25$

$\tan(25) = \frac{x}{87}$

$x = 87\tan(25^\circ)$

$x \approx 40.5687\ldots$ ft

2. Solve for h

$\tan\theta = \frac{opp}{adj}$

opp: h
adj = 87 ft
$\theta = 40^\circ$

$\tan(40^\circ) = \frac{h}{87}$

$h = 87\tan(40^\circ)$

$h \approx 73.001\ldots$ ft

3. Solve for y

$y = h - x$

$y = 87\tan(40^\circ) - 87\tan(25^\circ)$

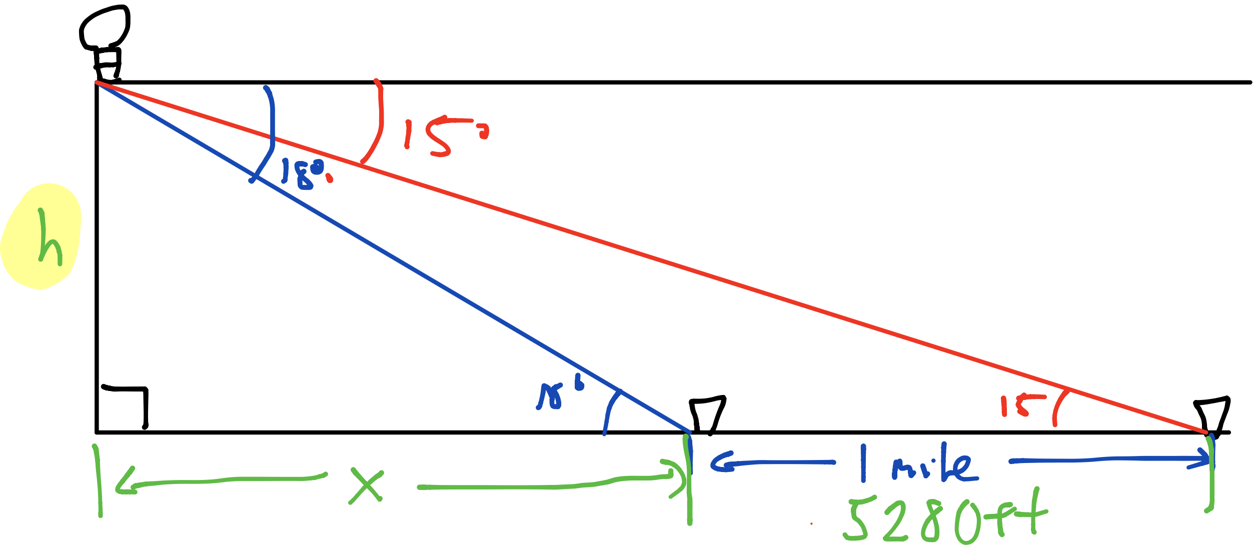
$y = 87\left(\tan(40^\circ) - \tan(25^\circ)\right)$

* Note: $\tan(40) - \tan(25) \neq \tan(15)$

$y \approx 32.4\ldots$ ft

$y \approx 32$ ft

A hot-air balloon is floating above a straight road. To calculate their height above the ground, the balloonists simultaneously measure the angle of depression to two consecutive mileposts on the road on the same side of the balloon. The angles of depression are found to be 15° and 18°. How high (in feet) is the ballon?



Our objective is to find h.

$$\tan(18^\circ) = \frac{h}{x}$$

$$h = x\tan(18)$$

$$\tan(15^\circ) = \frac{h}{x+5280}$$

$$h = (x+5280)\tan(15)$$

$$h = x\tan(15) + 5280\tan(15)$$

$$h = h$$

$$x\tan(18) = x\tan(15) + 5280\tan(15)$$

$$-x\tan(15) \quad -x\tan(15)$$

$$x\tan(18) - x\tan(15) = 5280\tan(15)$$

$$x\left(\tan(18) - \tan(15)\right) = 5280\tan(15)$$

$$\frac{x\,(\tan(18)-\tan(15))}{(\tan(18)-\tan(15))} = \frac{5280\tan(15)}{(\tan(18)-\tan(15))}$$

$$x = \frac{5280\tan(15)}{(\tan(18)-\tan(15))}$$

* Don't use calculator yet.

<u>We can solve for h now.</u>

$$h = x\tan(18)$$

$$h = \left(\frac{5280\tan(15)}{(\tan(18)-\tan(15))}\right)\cdot \frac{\tan(18)}{1}$$

$$h = \frac{5280\tan(15)\cdot\tan(18)}{\tan(18)-\tan(15)} \approx 8{,}069 \text{ ft}$$