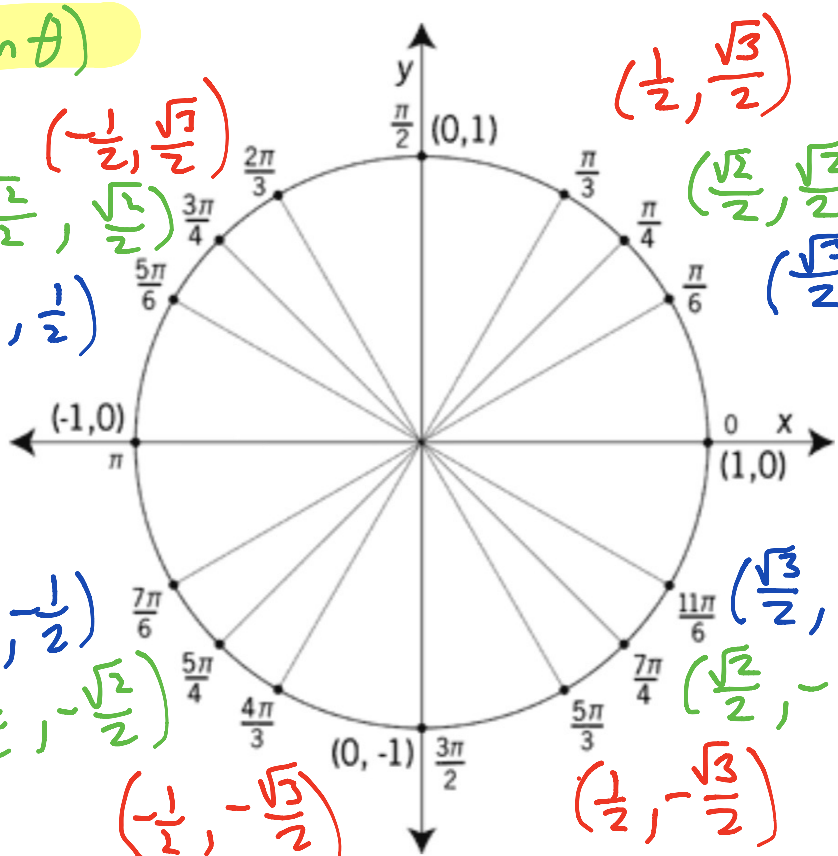
5. Find the coordinates for all the angles in the unit circle:

$(x,y) = (\cos\theta, \sin\theta)$



$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

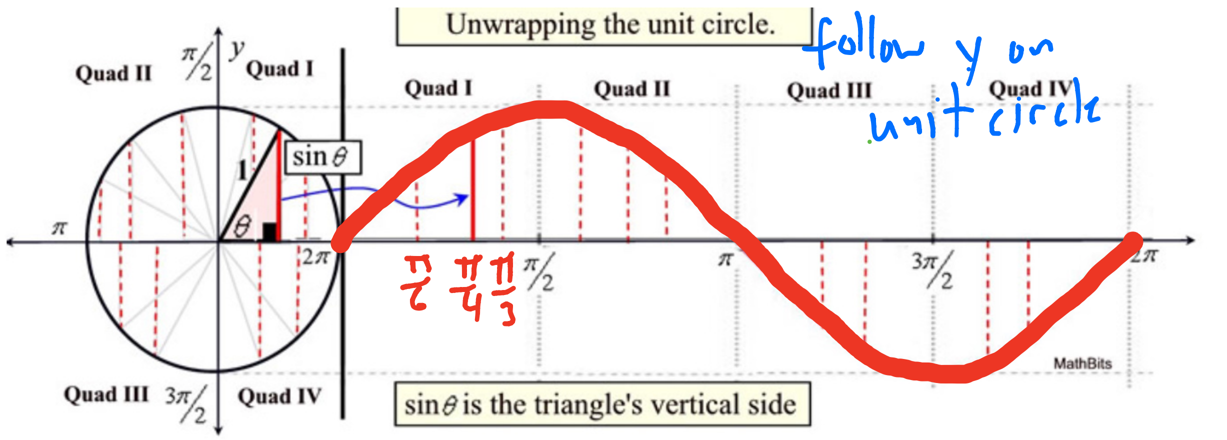
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

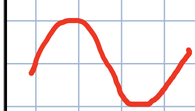
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

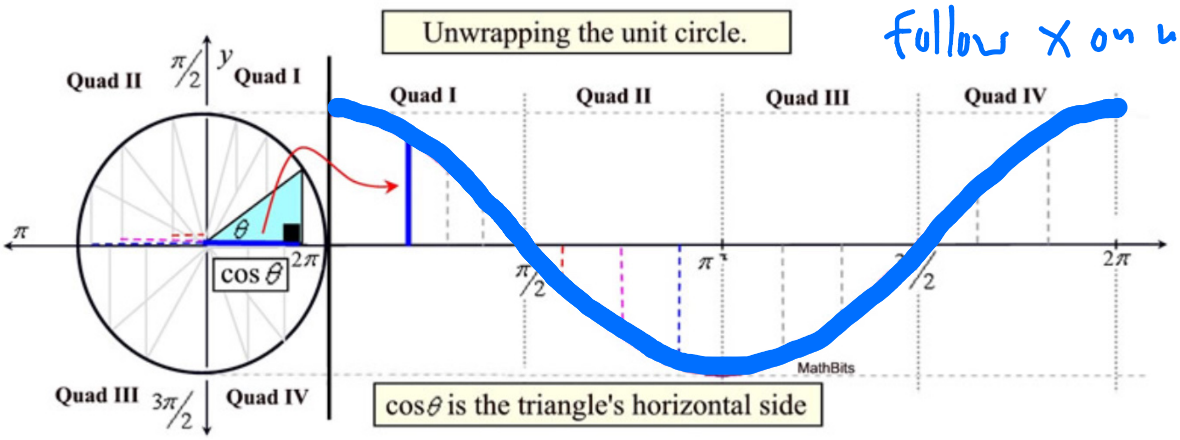
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



follow y on unit circle

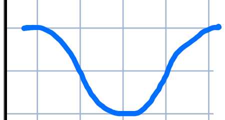
Graph of $y=\sin(x)$





follow x on unit circle

$y=\cos(x)$



Like the sine graph, the cosine graph will "repeat", making it a periodic function.

The graph will repeat every period of 2π.

Remember that cosine is negative in Quadrants II and III (the x-coordinates are negative).

$y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$

$|A|$ = amplitude

$y = D$ is the midline

* note: we don't do midlines in this class for some reason

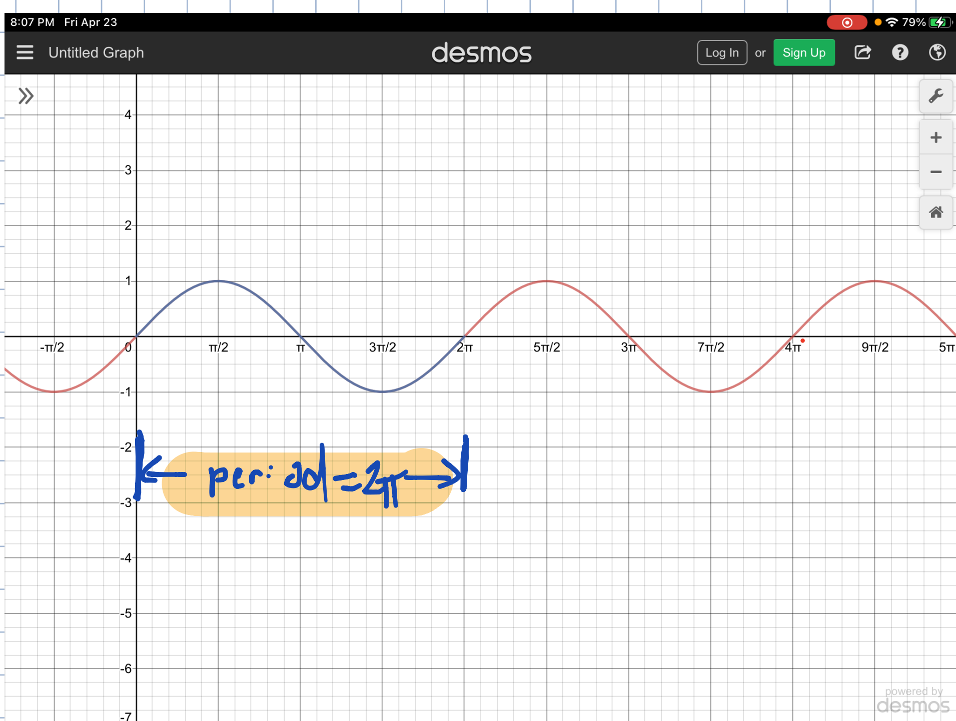
Period: $\dfrac{\text{normal period}}{|B|}$ ← how long before pattern repeats

* Note: - normal period for $\sin\theta$, $\cos\theta$, $\csc\theta$, $\sec\theta$ is 2π

- for $\tan\theta$, $\cot\theta$, normal period is π

Phase shift: $\dfrac{C}{B}$

* where the graph starts



$y = \sin(x)$

- repeats itself

max: 1

@ $x = \ldots, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$x = \frac{\pi}{2} + 2\pi n$

min: -1

@ $x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + 2\pi n$

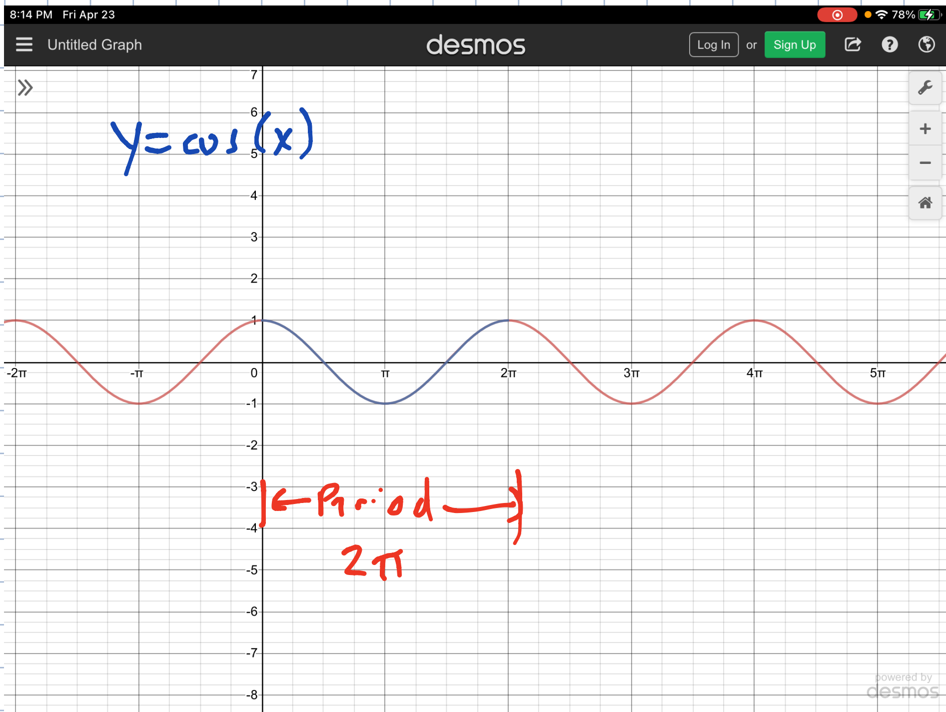
$n \in \mathbb{Z}$

$n \in \{0, \pm 1, \pm 2, \ldots\}$

Zeroes:, x-int @ $x = \ldots, 0, \pi, 2\pi, \ldots$

$x = \pi n$

Pattern: 0, max, 0, min, 0



max: 1

@ $x = 0, 2\pi, 4\pi, \ldots$

$x = 2n\pi$

"even multiples of π

min: -1

@ $x = \ldots, \pi, 3\pi, 5\pi, \ldots$

$x = (2n+1)\pi$

"odd multiples of π

zeroes:

@ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$x = \frac{\pi}{2} + 2\pi n$ or odd multiples of $\frac{\pi}{2}$

Pattern: max, 0, min, 0, max

Noticed that in both graphs that over the course of the period of 2 pi, the zeros, Max, min are located on a quarter of the interval. This implies that there is the rule of 1/4s for sine and cosine the graphs will follow the pattern listed for each.

$y = A\sin(Bx)$ or $y = A\cos(Bx)$

In MAT1375, we go deeper.

$y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$

Definitions

$y = D$ midline - horizontal centerline about which the function oscillates above and below.

- in our class $y = 0$ is midline

$|A|$ amplitude: $\frac{1}{2}$ positive distance between max and min

: distance from max/min to midline.

: if A is negative, positive function is reflected over midline.

Pattern

e.g.: $3\sin(x) \to 0$, max, 0, min, 0

$-3\sin(x) \to 0$, min, 0, max, 0

---

max/min swap when A is negative

$2\cos(x) \to$ max, 0, min, 0, max

$-2\cos(x) \to$ min, 0, max, 0, min

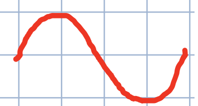
B frequency = number of cycles within a normal period length.

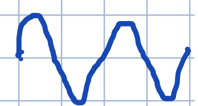
Normal period length: 2π

sin, cos, csc, sec functions

$\therefore \pi$

tan, cot functions

e.g. $\sin(1x)$ = one cycle over 2π 

$\sin(2x)$ = two cycles over 2π 

$\frac{\text{normal period}}{|B|}$

Period: the length of one cycle of a periodic function

horizontal / phase shift - horizontal translation of the graph

- starting point of the period.

Depends on the form

$y = Af(Bx - C) + D$, f is a trig function

$\rightarrow \frac{C}{B}$ → if $+$, shift $\rightarrow \frac{C}{B}$

→ if $-$, shift $\leftarrow \frac{C}{B}$

$x = \frac{C}{B}$ is the location of left most point of the cycle.

e.g. $y = 5\cos(2x)$

$y = A\cos(Bx - C)$

$A = 5$

$B = 2$

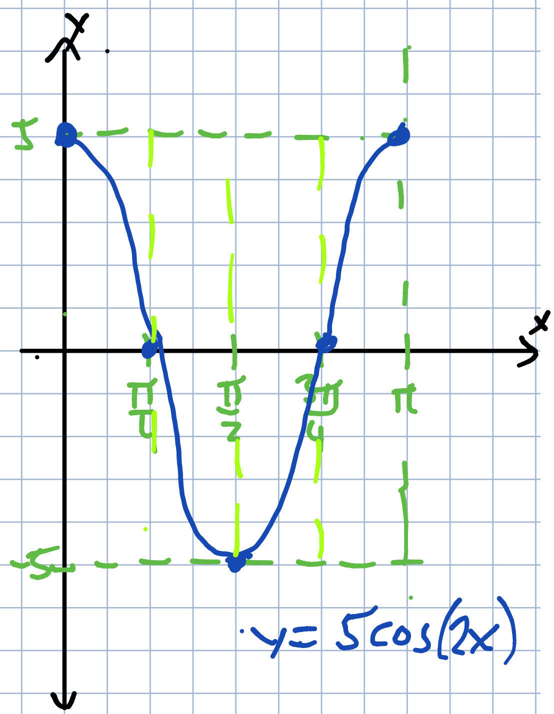
$C = 0$

amplitude: $|A| = 5$ → max: 5, min: −5

period length: $\dfrac{\text{normal period}}{|B|} = \dfrac{2\pi}{2} = \pi$

phase shift: $\dfrac{C}{B} = \dfrac{0}{2} = 0$

→ first point @ $x = 0$



last endpoint: phase shift + period

$0 + \pi = \pi$

→ last point @ $x = \pi$

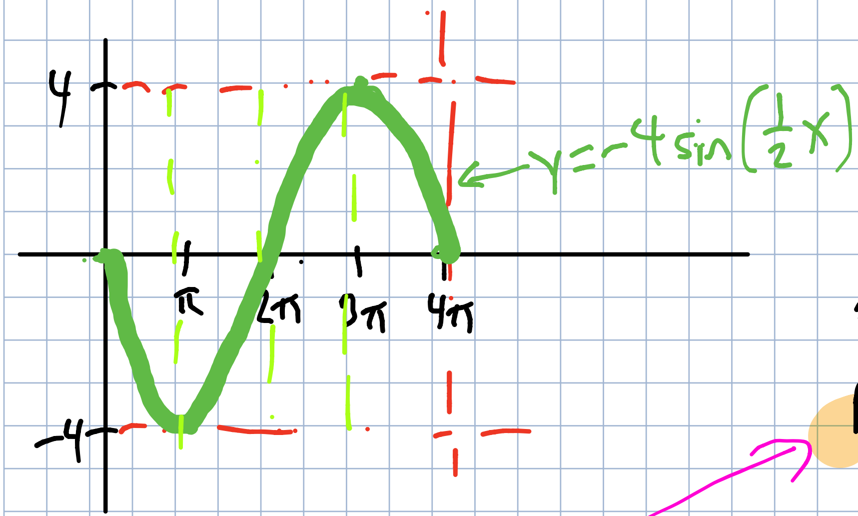
Rule of 4ths 1. divide period by 4 = $\dfrac{\pi}{4}$

2. phase shift + quarter of period

P	$x = 0$	max = 5
Q	$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$	0
R	$x = 0 + \frac{\pi}{4} + \frac{\pi}{4} = 0 + \frac{2\pi}{4} = \frac{\pi}{2}$	min: −5
S	$x = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$	0
T	$x = \pi$	max = 5

$P(0,5)$, $Q(\frac{\pi}{4},0)$, $R(\frac{\pi}{2},-5)$, $S(\frac{3\pi}{4},0)$, $T(\pi,5)$

$y = -4\sin\left(\frac{1}{2}x\right)$



$y = A\sin(Bx - C)$

$A = -4$ $\quad C = 0$

$B = \frac{1}{2}$

pattern is reflected

Amplitude: $|A| = |-4| = 4$

Period: $\frac{2\pi}{|B|} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$\frac{2\pi}{\frac{1}{2}} = 2\pi \div \frac{1}{2}$

$= 2\pi \cdot \frac{2}{1} = 4\pi$

Phase shift: $\frac{C}{B} = \frac{0}{\frac{1}{2}} = 0$

→ 1st point @ $x = 0$

Rule of 4ths: $\frac{\text{period}}{4} = \frac{4\pi}{4} = \pi$

Last point: phase shift + period

$0 + 4\pi = 4\pi$

P	$x = 0$	0
Q	$x = 0 + \pi = \pi$	min $= -4$
R	$x = 0 + 2\pi = 2\pi$	0
S	$x = 0 + 3\pi = 3\pi$	max $= 4$
T	$x = 4\pi$	0

$P(0, 0)$

$Q(\pi, -4)$

$R(2\pi, 0)$

$S(3\pi, 4)$

$T(4\pi, 0)$

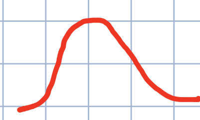
$Y = -2\cos\left(2x + \frac{\pi}{3}\right)$

$y = A\cos(Bx - C)$

$A = -2$

$B = 2$

$C = -\frac{\pi}{3}$



Amplitude: $|A| = |-2| = 2$

Phase shift: $\frac{C}{B} = \frac{-\frac{\pi}{3}}{2} = -\frac{\pi}{3} \cdot \frac{1}{2} = -\frac{\pi}{6}$

Period: $\frac{\text{normal period}}{|B|} = \frac{2\pi}{2} = \pi$

Last point: phase shift + period

$-\frac{\pi}{6} + \pi\left(\frac{6}{6}\right) = \frac{5\pi}{6}$

Rule of 4ths: 1. $\frac{\text{Period}}{4} = \frac{\pi}{4}$

First point + $\frac{\text{period}}{4}$

P: $x = -\frac{\pi}{6}$

Q $x = -\frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi}{12}$

R $x = -\frac{\pi}{6} + \frac{2\pi}{4} = \frac{\pi}{3}$

S $x = -\frac{\pi}{6} + \frac{3\pi}{4} = \frac{7\pi}{12}$

T: $x = \frac{5\pi}{6}$