Unit circle – circle with radius of 1

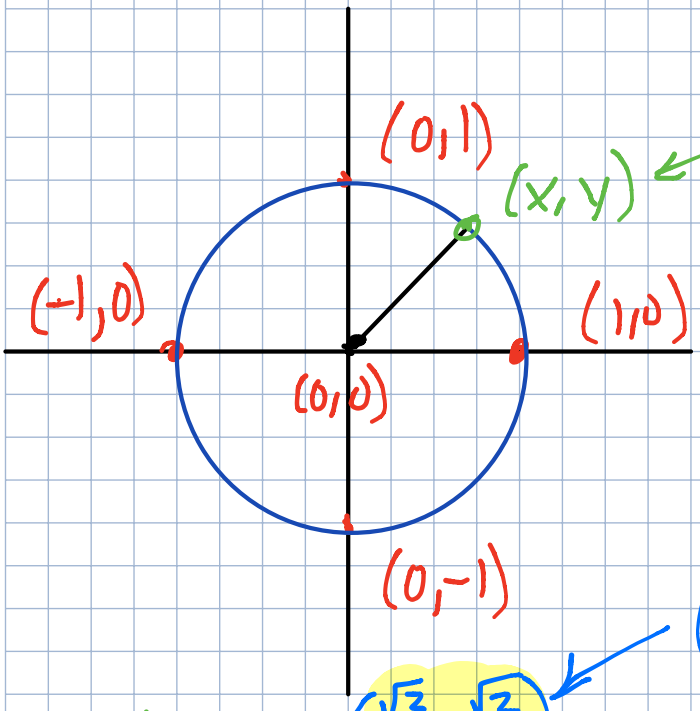
– For today's lesson, center is (0,0)

– Equation of unit circle.

$(x-h)^2+(y-k)^2=r^2$, (h,k) is center, r = radius

$(x-0)^2+(y-0)^2=(1)^2$

$x^2+y^2=1$

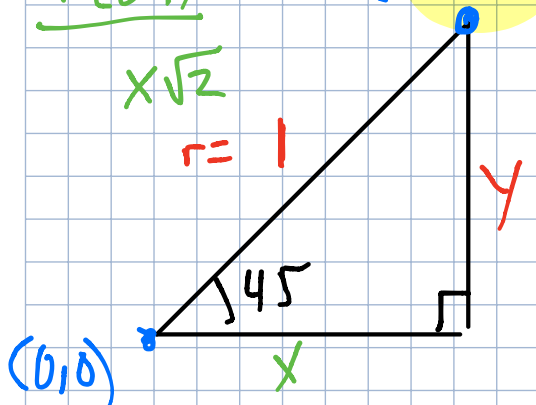


$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ $(\cos(45), \sin(45))$

Recall

$x\sqrt{2}$

$r=1$



$\cos(45^\circ)=\frac{adj}{hyp}$ or $\sin(45)=\frac{opp}{hyp}$

$\cos(45)=\frac{x}{1}$ $\sin(45)=\frac{y}{1}$

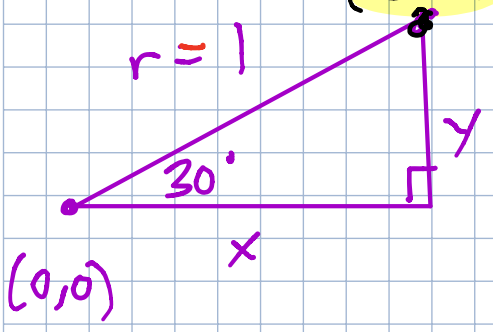
$\cos(45^\circ)=x$ $\sin(45)=y$

$x=y$ because isosceles

$\frac{\sqrt{2}}{2}=x$ $\frac{\sqrt{2}}{2}=y$

$(\cos(30), \sin(30))$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$\cos(30) = \frac{adj}{hyp}$

$\cos(30) = \frac{x}{1}$

$\cos(30) = x$

$\frac{\sqrt{3}}{2} = x$

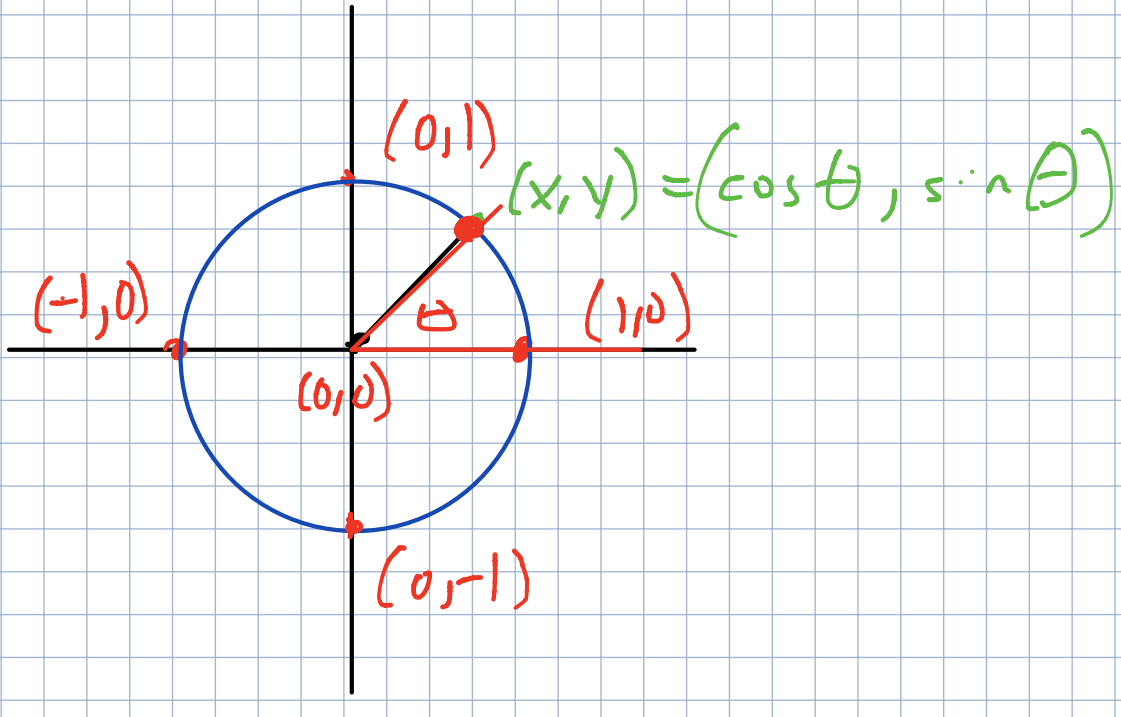
$\sin(30) = \frac{opp}{hyp}$

$\sin(30) = \frac{y}{1}$

$\sin(30) = y$

$\frac{1}{2} = y$

On a unit circle, $(x, y) = (\cos\theta, \sin\theta)$

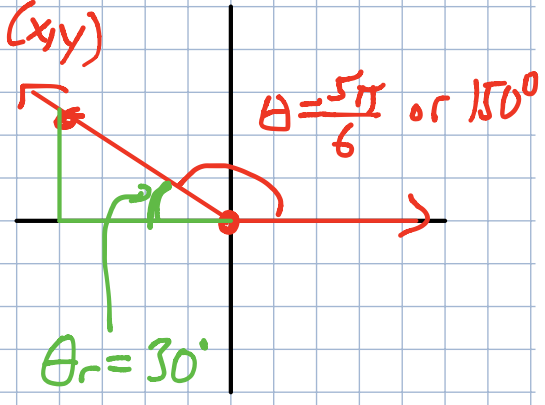


* Very important to remember
  - trig ratios of special angles
    $\{0, 30, 45, 60, 90 \ldots\}$
  - radian measure.

Find points on unit circle associated with rotation θ.

Consider $\frac{5\pi}{6}$ radians

$\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150^\circ$



$\theta_r = 180^\circ - 150^\circ$

QII $(x,y) = (-,+)$

$(x,y) = (\cos\theta, \sin\theta)$

Working from degrees

$= (\cos(150^\circ), \sin(150^\circ))$

Note: we have never worked with trig ratios of angles greater than 90°

$\underset{QII}{=} (-\cos(\theta_r), +\sin(\theta_r))$

$= (-\cos(30^\circ), \sin(30^\circ))$

$= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Working in radians

$(x,y) = \left(\cos\left(\frac{5\pi}{6}\right), \sin\left(\frac{5\pi}{6}\right)\right)$

QII $= (-\cos\theta_r, +\sin\theta_r)$

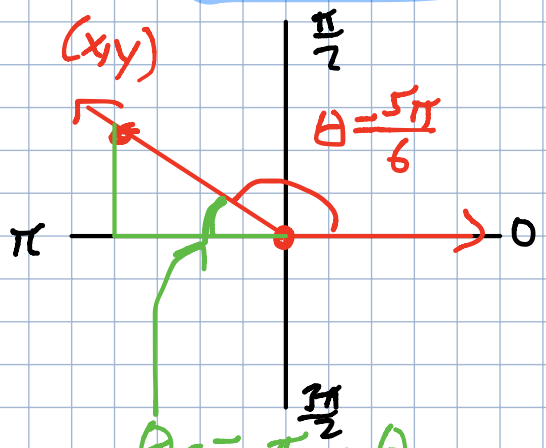
$= \left(-\cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)\right)$

Recall $\frac{\pi}{6}$ radians $= 30°$

$= (-\cos(30), \sin(30))$

$= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

QII (−, +)



$\theta_r = \pi - \theta$

$= \pi - \frac{5\pi}{6}$

$= \frac{6\pi}{6} - \frac{5\pi}{6}$

$\theta_r = \frac{\pi}{6}$

* Note: reference angle in radians only.

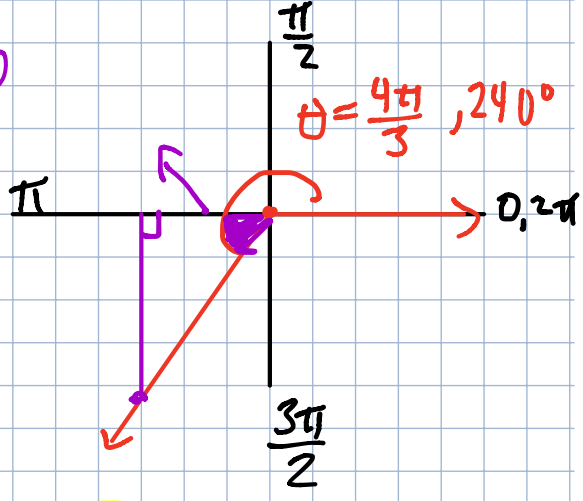
Let $\theta = \frac{4\pi}{3}$

$\pi + \theta_r = \theta$

$\theta_r = \theta - \pi$

$\theta_r = \frac{4\pi}{3} - \frac{3\pi}{3}$

$\theta_r = \frac{\pi}{3}$



$(x, y) = (\cos\theta, \sin\theta)$

$= \left(\cos\left(\frac{4\pi}{3}\right), \sin\left(\frac{4\pi}{3}\right)\right)$

QIII $= \left(-\cos\left(\frac{\pi}{3}\right), -\sin\left(\frac{\pi}{3}\right)\right)$

QIII $\Rightarrow (-, -)$ x, y

*Recall $\frac{\pi}{3} = 60°$

$= \left(-\cos(60°), -\sin(60°)\right)$

$= \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

---

* Only applies for special angles in radians

$\theta = \frac{n\pi}{6}, \frac{n\pi}{4}, \frac{n\pi}{3}$, where $n \in \mathbb{Z} \setminus \{0\}$

$n \in \{\pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$

nonzero integer

$\theta_r = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$

Try $\theta = \frac{7\pi}{4}$

QIV $(+,-)$

$\theta_r = \frac{\pi}{4}$

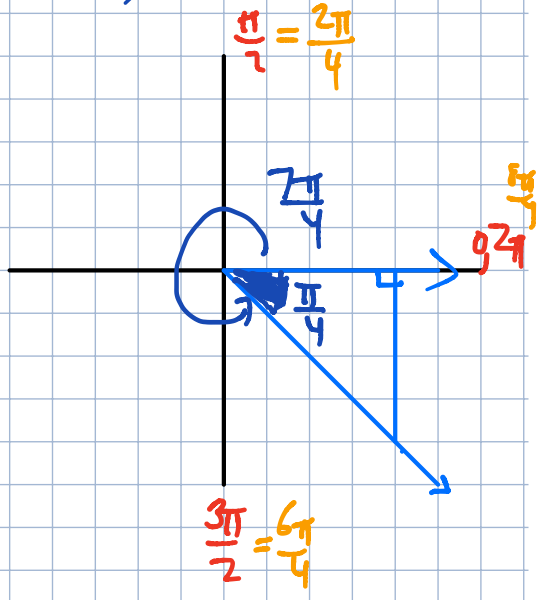
$(x,y) = (\cos\theta, \sin\theta)$

$= \left(+\cos\left(\frac{\pi}{4}\right), -\sin\left(\frac{\pi}{4}\right)\right)$

QIV

$= (+\cos(45), \sin(45))$

$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$



Evaluate six trig functions of unit circle

Equation: $x^2 + y^2 = 1^2$ $\quad (x,y) = (\cos\theta, \sin\theta)$

$\cos\theta = x$ $\quad \sec\theta = \frac{1}{x}$

$\sin\theta = y$ $\quad \csc\theta = \frac{1}{y}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$