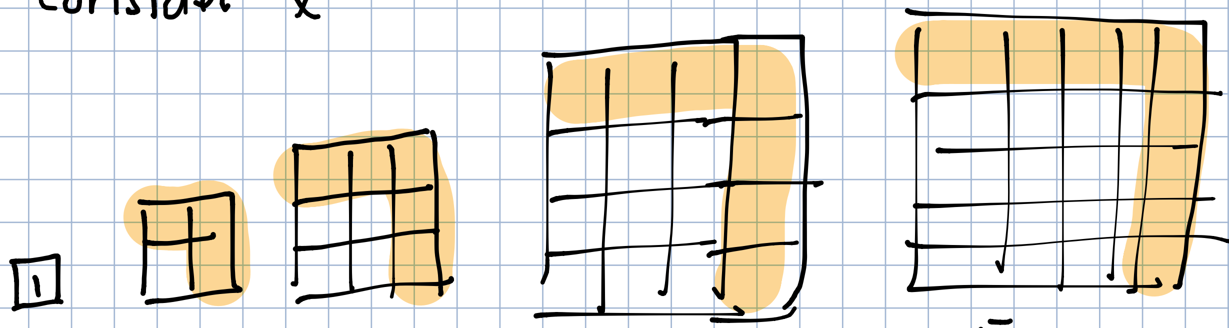
Consider x^2



Let $x = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$-1 \quad -2 \quad -3 \quad -4 \quad -5 \quad -6$

$x^2 = 1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36$

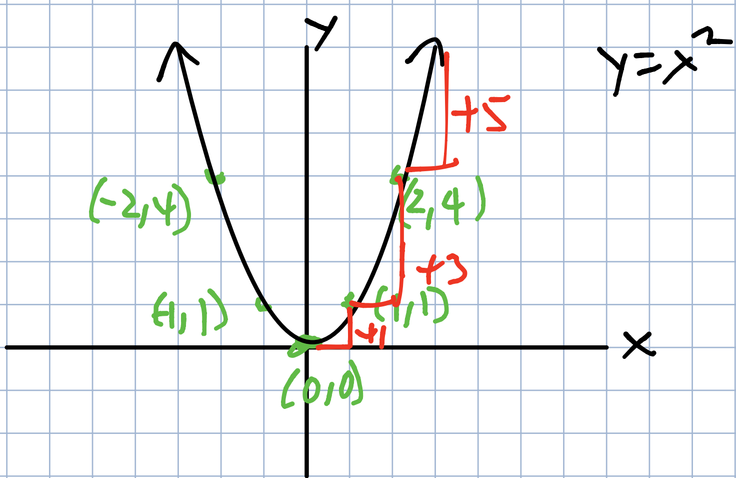
$+3 \quad +5 \quad +7 \quad +9 \quad +11$

$+2 \quad +2 \quad +2 \quad +2$

$$x^2 = \sum_{n=0}^{x} (2n+1)$$

→ The sum of the sequence of odd numbers

sum from 0 to x



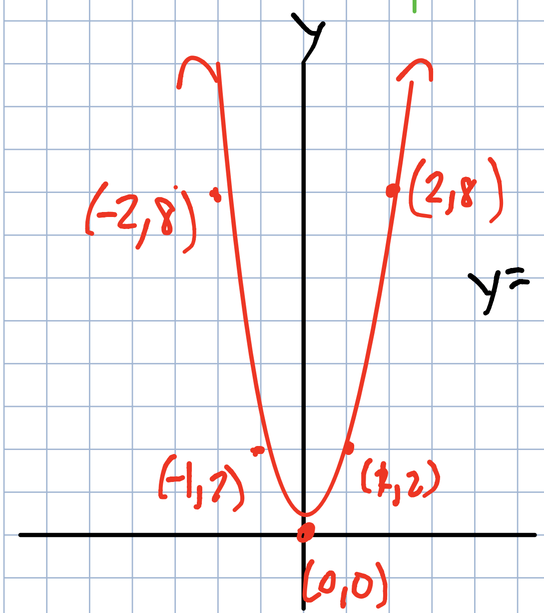
$y = 2x^2$

x	0	± 1	± 2	± 3	± 4
$2x^2$	0	2	8	18	32

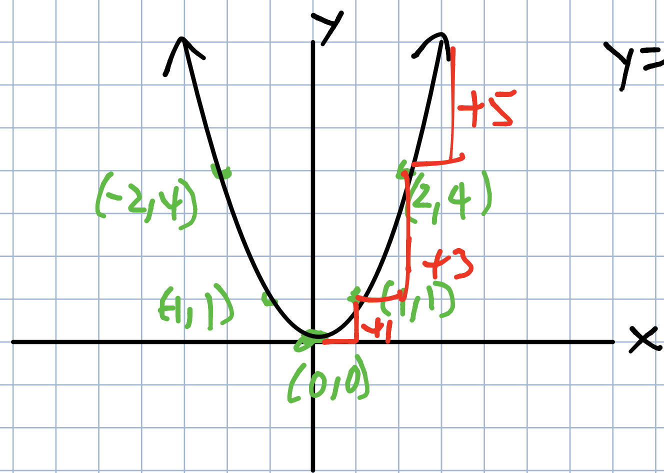
+2 +6 +10 +14

+4 +4 +4

$2 = 2 \cdot 1$
$6 = 2 \cdot 3$
$10 = 2 \cdot 5$
$14 : 2 \cdot 7$



$y = 2x^2$

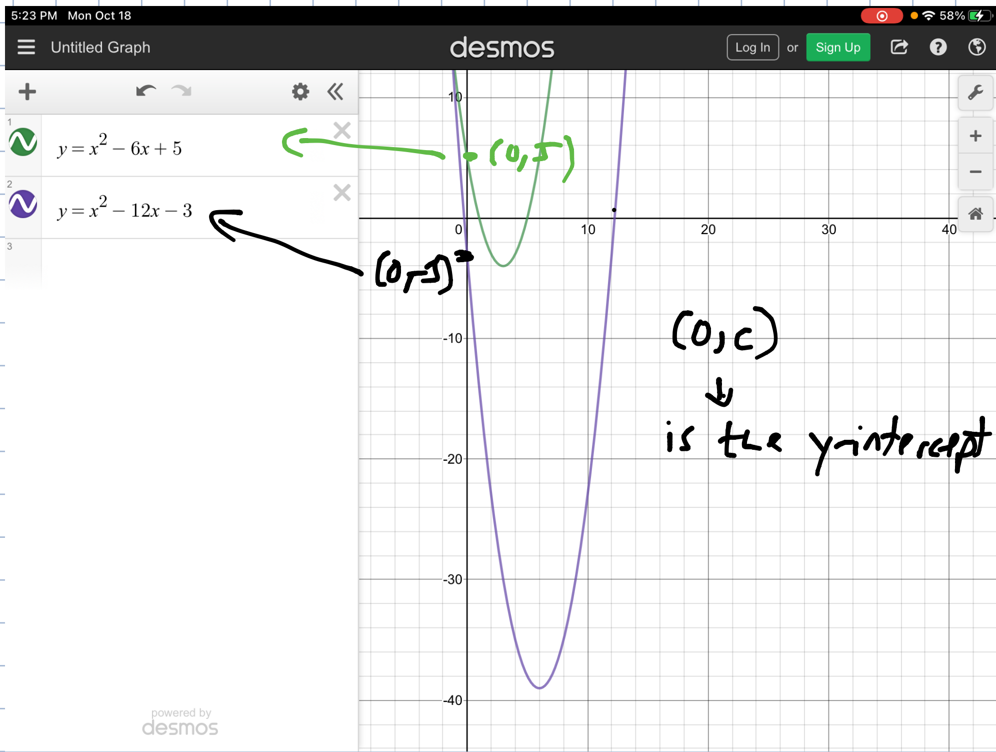


$y = x^2$

Given: $y = ax^2$

* if $|a|$ is larger, parabola is sharper.
* if $|a|$ is smaller, parabola is wider.
* if a is negative, parabola is reflected over x-axis

Consider the graph. $y=ax^2+bx+c$

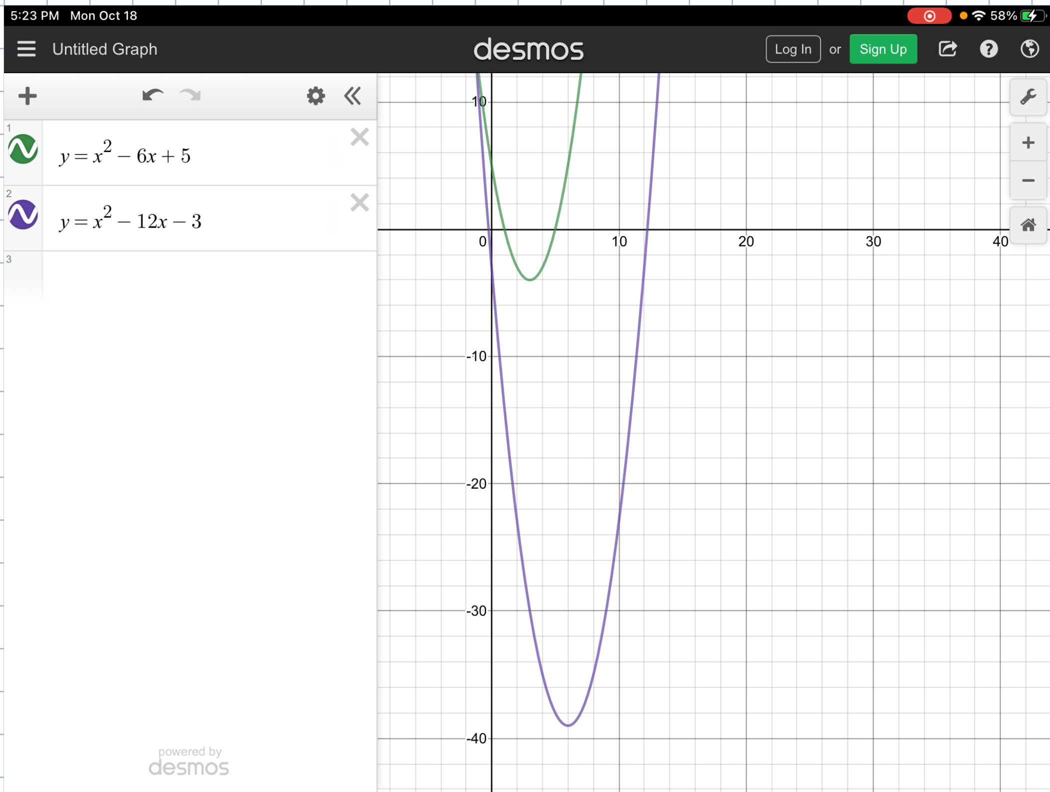


* In standard form, you can only identify y-intercept

$y=ax^2+bx+c \rightarrow$ y-int is $(0,c)$

Recall $y=mx+b \rightarrow$ y-int is $(0,b)$

* To find y-intercept always set x to 0.



$y = x^2 - 6x + 5$

$y = (x-1)(x-5)$ To find x-int

Let $y = 0$: $(x-1)(x-5) = 0$

$$\begin{array}{ccc} x - 1 = 0 & \text{or} & x - 5 = 0 \\ +1 \quad +1 & & +5 \quad +5 \\ \hline x = 1 & & x = 5 \end{array}$$

$$\boxed{(1, 0) \qquad (5, 0)}$$ x-intercepts

$y = x^2 - 12x - 3$ ← Not factorable

To find x-int, let $y = 0$

$$x^2 - 12x - 3 = 0$$
$$\quad +3 \quad +3$$

$$x^2 - 12x = 3$$

$$x^2 - 12x + \left(-\frac{12}{2}\right)^2 = 3 + \left(-\frac{12}{2}\right)^2$$

$$(x-6)^2 = 3 + 36$$

$$(x-6)^2 = 39$$

$$x - 6 = \pm\sqrt{39}$$

$$x = 6 \pm \sqrt{39} \nearrow \begin{array}{l} 6+\sqrt{39} \\ 6-\sqrt{39} \end{array}$$

x-intercepts: $(6-\sqrt{39}, 0)$, $(6+\sqrt{39}, 0)$

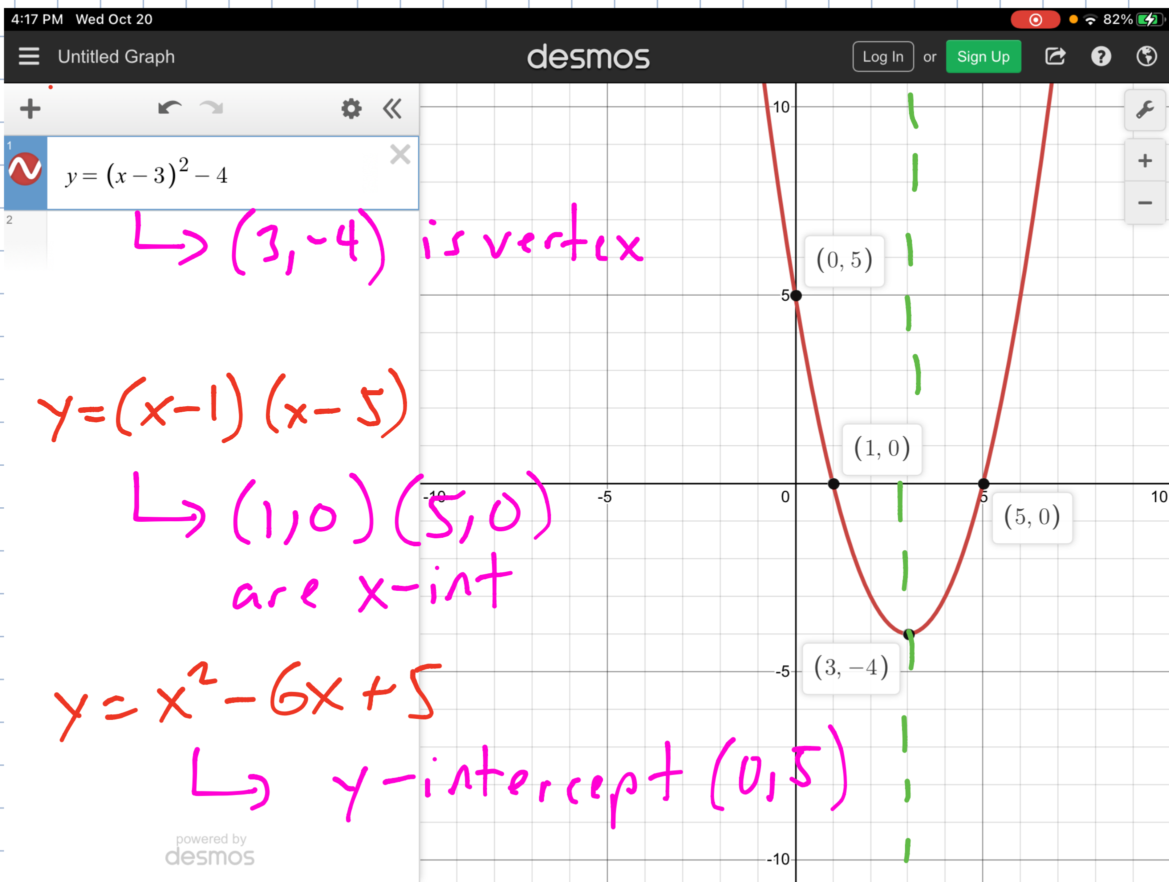
* DO NOT GIVE A DECIMAL ANSWER

* roots form of quadratic

$$y = a(x - r_1)(x - r_2)$$

x-intercepts are $(r_1, 0)$, $(r_2, 0)$

Consider the case of $y = a(x-h)^2 + k$



↳ $(3,-4)$ is vertex

$y = (x-1)(x-5)$

↳ $(1,0)$ $(5,0)$ are x-int

$y = x^2 - 6x + 5$

↳ y-intercept $(0,5)$

$y = (x-3)^2 - 4$ $(3,-4)$ is vertex

$x = 3$ axis of symmetry

middle line of reflection

-4 'lowest point' ← min

extremum

$y=a(x-h)^2+k$ vertex is (h,k)

$x=h$ is axis of symmetry

$k=$ max/min value of the quadratic

$y=(x-3)^2-4$ looks very similar to $(x-3)^2-4=0$

* $x^2-6x+5=0 \rightarrow (x-3)^2-4=0$

* complete the square

$$\begin{array}{rcl} y &=& x^2-6x+5 \\ -5 & & -5 \\ \hline y-5 &=& x^2-6x \end{array} \longrightarrow y=(x-3)^2-4$$

$y-5+9=x^2-6x+\left(-\frac{6}{2}\right)^2$

$$\begin{array}{rcl} y+4 &=& (x-3)^2 \\ -4 & & -4 \\ \hline y &=& (x-3)^2-4 \end{array}$$

$y=(x-3)^2+(-4)$

$y=a(x-h)^2+k$

vertex: $(3,-4)$

$y = x^2 - 10x + 20$

Standard form

y-int: $(0, c)$

x-intercepts

y-intercepts $(0, 20)$

vertex:

complete the square to find the vertex

$$y = x^2 - 10x + 20$$

$$y - 20 = x^2 - 10x$$

$$y - 20 + 25 = x^2 - 10x + \left(-\frac{10}{2}\right)^2$$

$$y + 5 = (x - 5)^2$$

$$-5 \qquad -5$$

$$y = (x-5)^2 - 5$$

$$y = (x - 5)^2 + (-5)$$

$y = a(x-h)^2 + k$

vertex: $(5, -5)$

$y = x^2 - 10x + 20 \leftarrow$ not factorable

x-int

$$x^2 - 10x + 20 = 0$$

CTS

$$(x-5)^2 - 5 = 0$$

by QF

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{rl} & +5 \quad +5 \\ \hline (x-5)^2 & = 5 \end{array}$$

$$\begin{array}{rl} x-5 & = \pm\sqrt{5} \\ +5 & \quad +5 \\ \hline x & = 5 \pm \sqrt{5} \end{array}$$

x-int: $(5+\sqrt{5}, 0)$ $(5-\sqrt{5}, 0)$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100-80}}{2}$$

$$x = \frac{10}{2} \pm \frac{\sqrt{20}}{2}$$

$$x = 5 \pm \frac{\cancel{\sqrt{4}}\sqrt{5}}{\cancel{2}}$$

$$x = 5 \pm \sqrt{5}$$

x-int: $(5+\sqrt{5}, 0)$ $(5-\sqrt{5}, 0)$

$$y = 2x^2 + 8x + \underline{\underline{39}}$$

x-int:

y-int: (0,39)

vertex: (-2,31)

$$y - 39 = 2x^2 + 8x$$

$$y - 39 = 2(x^2 + 4x)$$

$$\frac{y}{2} - \frac{39}{2} = \frac{\cancel{2}(x^2 + 4x)}{\cancel{2}}$$

$$\frac{y}{2} - \frac{39}{2} + 4 = x^2 + 4x + \left(\frac{4}{2}\right)^2$$

$$\frac{y}{2} - \frac{39}{2} + 4\left(\frac{2}{2}\right) = x^2 + 4x + \left(\frac{4}{2}\right)^2$$

$$\frac{y}{2} - \frac{39}{2} + \frac{8}{2} = (x + 2)^2$$

$$\frac{y}{2} - \frac{31}{2} = (x + 2)^2$$

$$\frac{y}{2} = (x+2)^2 + \frac{31}{2}$$

$$\cancel{2}\frac{y}{\cancel{2}} = 2\left((x+2)^2 + \frac{31}{2}\right)$$

$$y = a(x-h)^2 + k$$

$$y = 2(x+2)^2 + 2 \cdot \frac{31}{2}$$

$$y = 2(x+2)^2 + 31$$

vertex:

(-2, 31)

x-int: $y=0$

$$2(x+2)^2+31=0$$
$$\underline{\quad -31 \quad -31}$$
$$2(x+2)^2 = -31$$

$$(x+2)^2 = -\frac{31}{2}$$

$$x+2 = \pm\sqrt{-\frac{31}{2}}$$

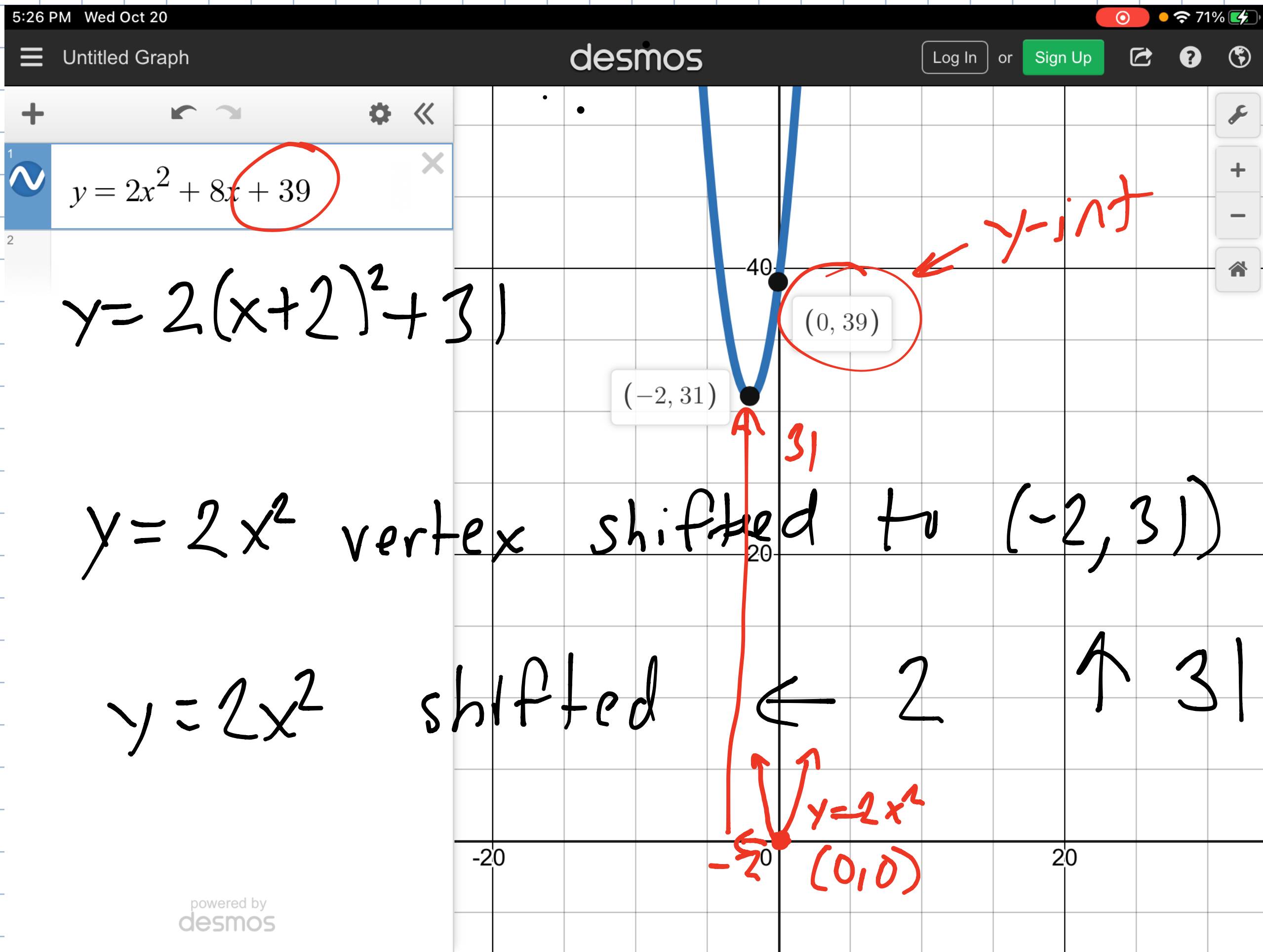
$$x+2 = \pm\frac{\sqrt{31}}{\sqrt{2}}\,i \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$x+2 = \pm\frac{\sqrt{62}}{2}\,i$$

$$x = -2 \pm \frac{\sqrt{62}}{2}\,i$$

x-int?: $\left(-2+\frac{\sqrt{62}}{2}i,\ 0\right), \left(-2-\frac{\sqrt{62}}{2}i,\ 0\right)$

Since $ax^2+bx+c=0$ resulted in x being a complex number, no x-int exist.



$y = 2(x+2)^2 + 31$

$y = 2x^2$ vertex shifted to $(-2, 31)$

$y = 2x^2$ shifted $\leftarrow 2$ $\uparrow 31$

$y = 2x^2 + 8x + 39$

by completing the square

$y = 2(x+2)^2 + 31$

vertex: $(-2, 31)$

$$ax^2 + bx + c = y$$
$$\qquad\qquad -c \quad -c$$

$$ax^2 + bx = y - c$$

$$\frac{ax^2 + bx}{a} = \frac{y - c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Perfect square

⋮

$$y = a\left(x - \left(-\frac{b}{2a}\right)\right) +$$

vertex: (h, k). $h = -\frac{b}{2a}$

$$y = 2x^2 + 8x + 39$$

$$h = -\frac{b}{2a} = -\frac{(8)}{2(2)} = -\frac{8}{4} = -2$$

$h = -2$, $x = -2$ axis of symmetry

$$k = 2(-2)^2 + 8(-2) + 39$$

$$= 2(4) + 8(-2) + 39$$

$$= 8 - 16 + 39$$

$$= -8 + 39 = 31$$

$k = 31$

vertex (h, k): $(-2, 31)$