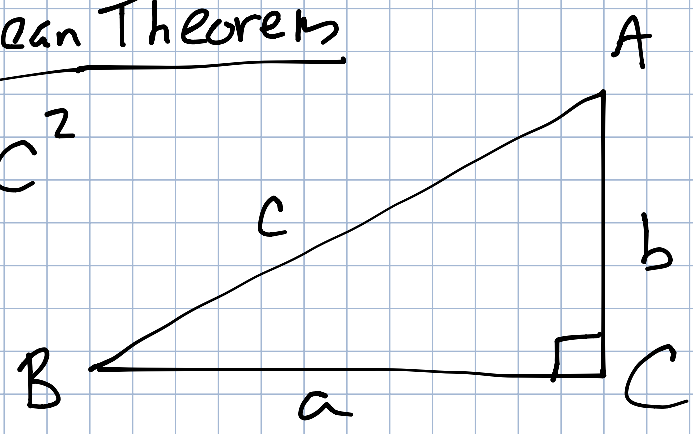
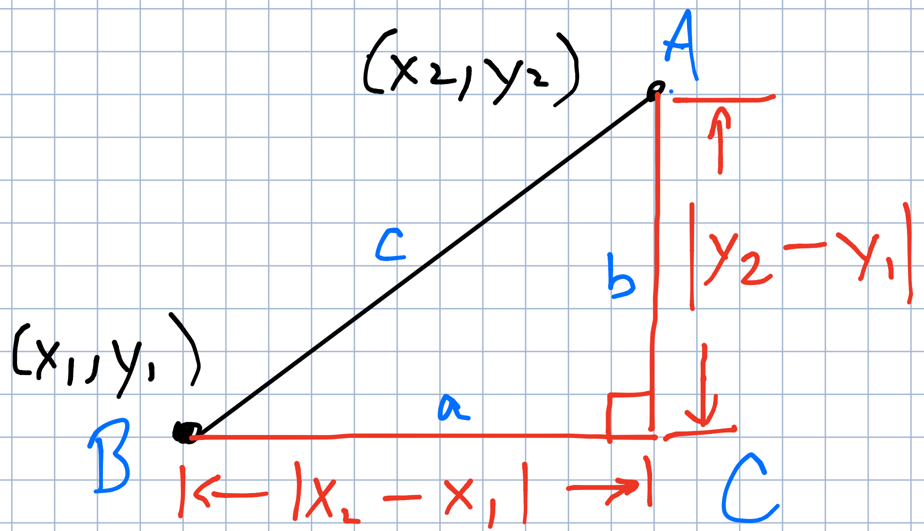
Recall Pythagorean Theorem

$$a^2 + b^2 = c^2$$



Given $\triangle ABC$ where $\angle C$ is a right angle,
then $a^2+b^2=c^2$



distance between (x_1, y_1) and (x_2, y_2)

*created right $\triangle ABC$ with $m(\angle C) = 90°$

$$a^2 + b^2 = c^2 \quad \text{(Pythagorean Theorem)}$$

$$(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 = c^2$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$$

$$\pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = c$$

* distance cannot be negative

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

* Δ means change

Find distance between 2 points $(6, -3)$ and $(1, -8)$

(x_1, y_1) (x_2, y_2)

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{((1) - (6))^2 + ((-8) - (-3))^2}$$

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$$

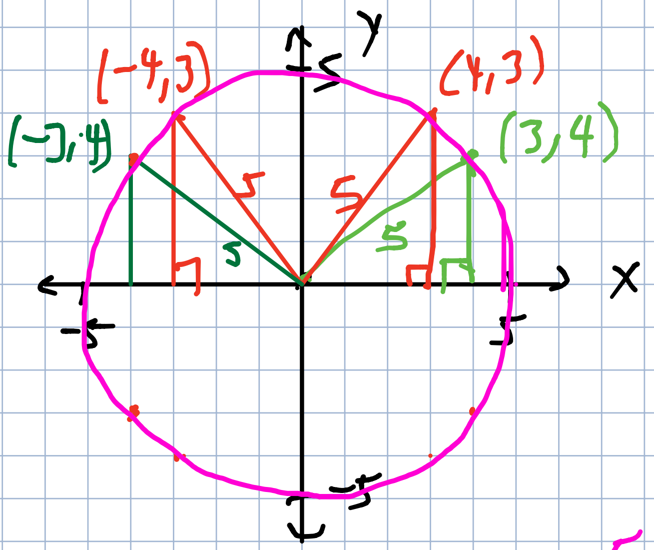
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ ← if in 3D.

Find the midpoint $(6,-3)$ and $(1,-8)$

(x_1, y_1) (x_2, y_2)

$$\text{mdpt} = \left(\frac{6+1}{2}, \frac{-3+-8}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{-11}{2}\right)$$



radius = 5

Pythagorean Triple:

a set of integers which complete the Pythagorean Theorem

e.g., 3-4-5

$$(3)^2+(4)^2=(5)^2$$

* If we keep drawing right triangles on (x,y) axis, then vertices not on an axis will form a circle.

$a^2+b^2=c^2 \longrightarrow x^2+y^2=r^2$

Pythagorean Theorem

(x,y) = point on a circle

r = radius

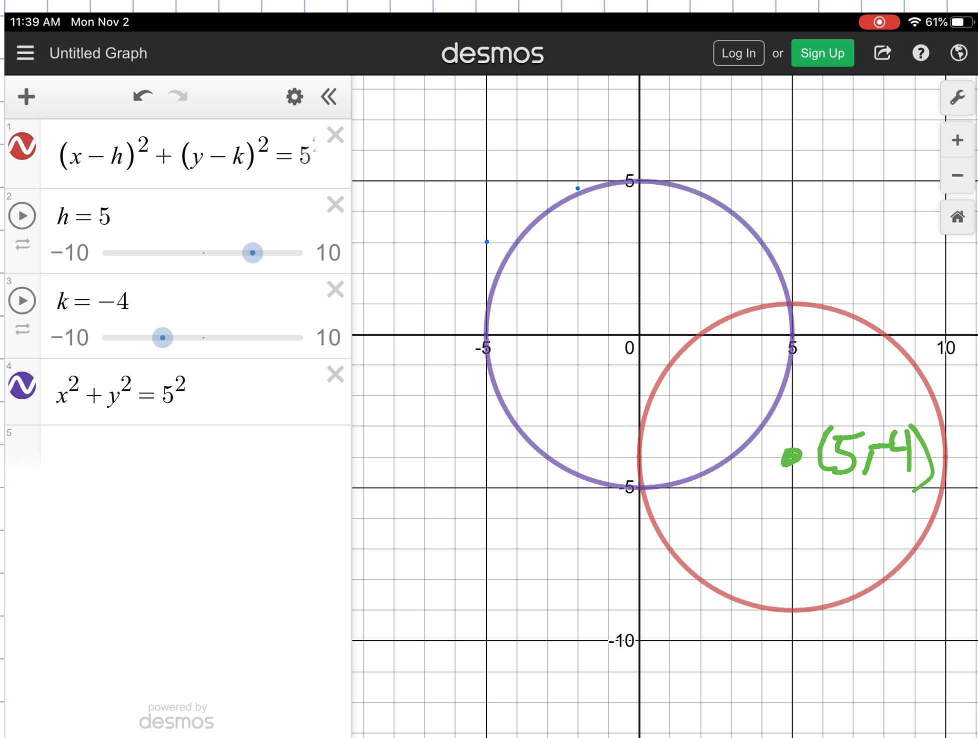
center is $(0,0)$

* Euclid's 3rd Postulate

Given any straight line segment, a circle can be drawn, having the segment as a radius and one endpoint as the center.

Standard form for the equation of a circle

$$(x-h)^2+(y-k)^2=r^2 \qquad r = \text{radius}$$



Given $(x-5)^2+(y-(-4))^2=5^2$

radius $=5$

center $(h,k)=(5,-4)$

$(x-h)^2+(y-k)^2=r^2$

$r=$ radius

$(h,k)=$ center

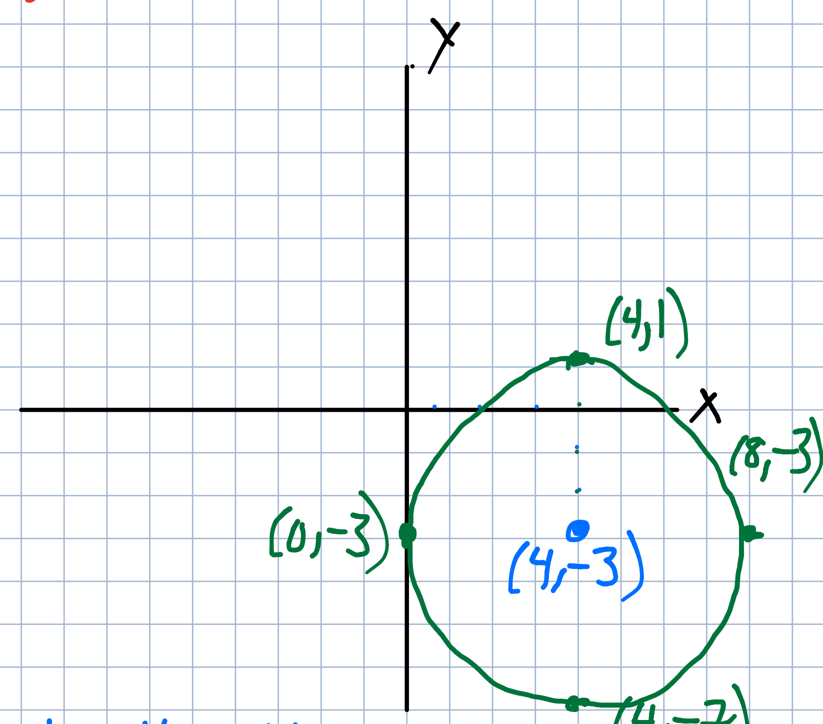
$(x-h)^2+(y-k)^2=r^2$

$(x-4)^2+(y+3)^2=16$

$(x-4)^2+(y-(-3))^3=(4)^2$

radius $=\sqrt{16}=4$

center $=(4,-3)$



* on final, want 4 points other than center

Since radius is 4, pick points 4 left from center.
right
up
down

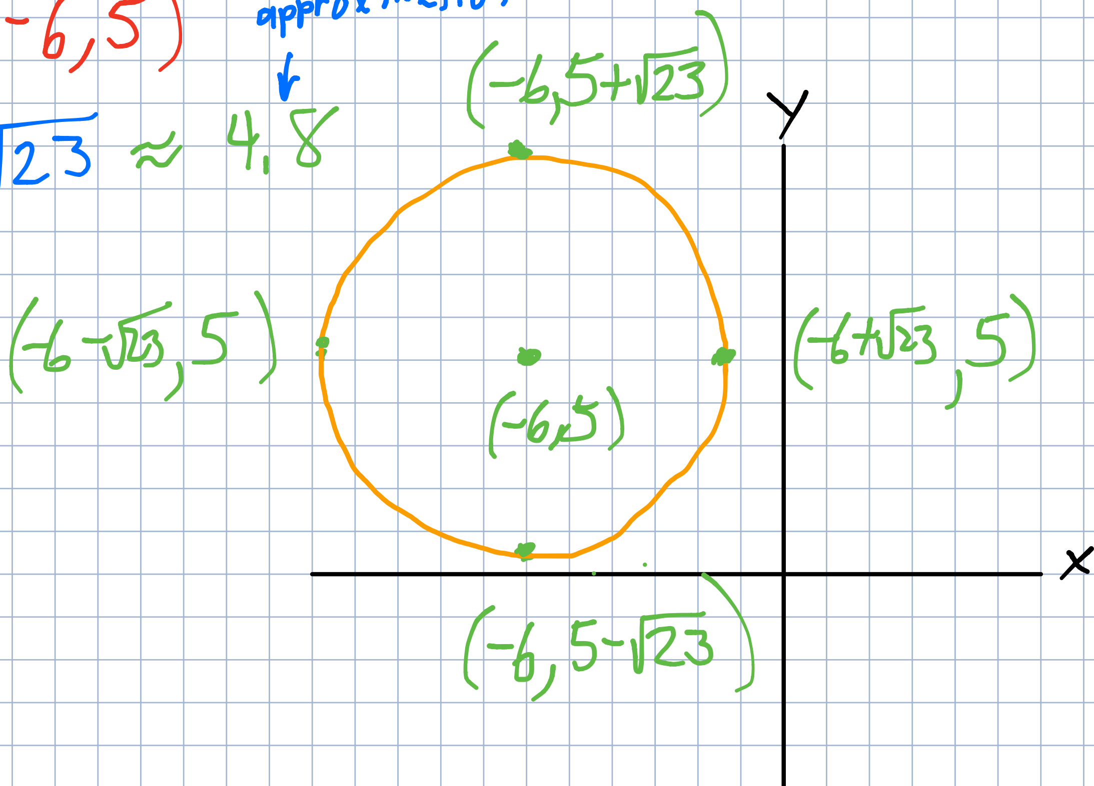
$$(x+6)^2+(y-5)^2=23$$

$$(x-(-6))^2+(y-5)^2=(\sqrt{23})^2$$

center: $(-6,5)$

approximation

radius: $\sqrt{23} \approx 4.8$



Rewrite the equation for the circle in standard form.

$x^2 + 12x + y^2 + 4y + 36 = 0 \rightarrow (x-h)^2 + (y-k)^2 = r^2$

should indicate that this is a circle.

$$\begin{aligned} x^2 + 12x + y^2 + 4y + 36 &= 0 \\ -36 &\quad -36 \\ \hline x^2 + 12x + y^2 + 4y &= -36 \end{aligned}$$

"double quadratic, two variables"

$$(x^2 + 12x) + (y^2 + 4y) = -36$$

$$\left(x^2 + 12x + \left(\frac{12}{2}\right)^2\right) + \left(y^2 + 4y + \left(\frac{4}{2}\right)^2\right) = -36 + \left(\frac{12}{2}\right)^2 + \left(\frac{4}{2}\right)^2$$

$$(x^2 + 12x + 36) + (y^2 + 4y + 4) = \cancel{-36 + 36} + 4$$

$$(x+6)^2 + (y+2)^2 = 4$$

$$(x-(-6))^2 + (y-(-2))^2 = (2)^2$$

center: $(-6, -2)$

radius: $\sqrt{4} = 2$

A: $(-13, -7)$
B: $(-3, 17)$

$\overline{AB}$ is the diameter of the circle

* Find equation of the circle

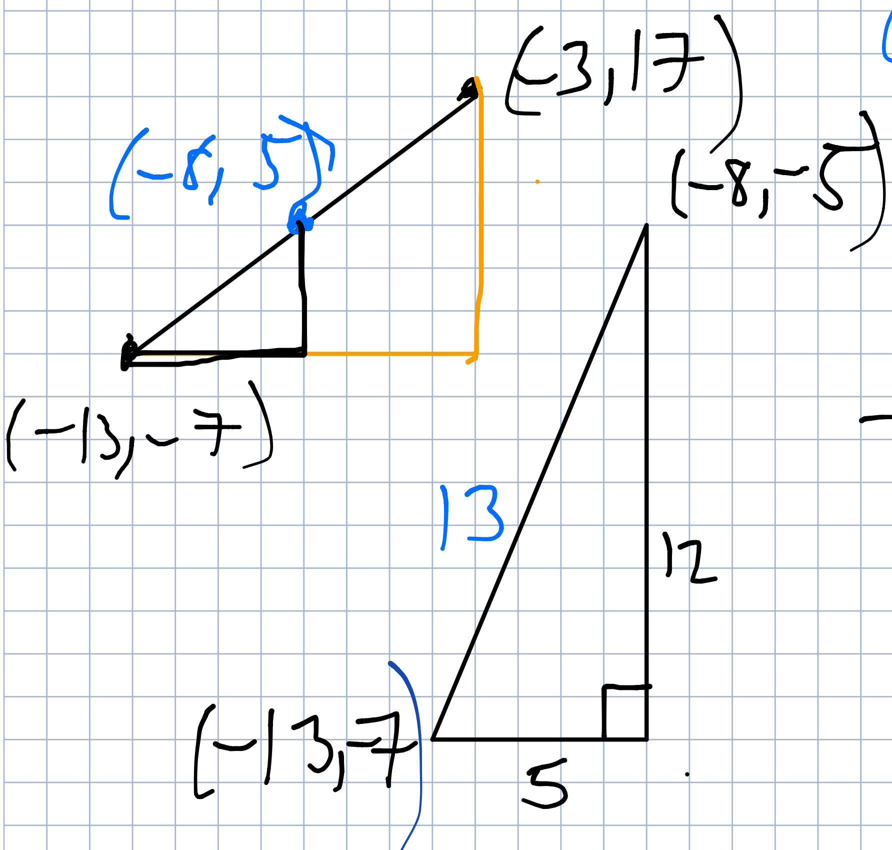
radius: ? center: ?

center: midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{(-13)+(-3)}{2}, \frac{(-7)+(17)}{2}\right)$

$\left(\frac{-16}{2}, \frac{10}{2}\right) = (-8, 5)$

radius:



$(-8)-(-13) =$
$-8+13 = 5$

$5-(-7) =$
$5+7 = 12$

$5^2+12^2 = r^2$
$25+144 = r^2$
$169 = r^2$
$13 = r$

center: $(-8, 5)$ radius: 13

$$(x+8)^2 + (y-5)^2 = 13^2$$

$$(x+8)^2 + (y-5)^2 = 169$$

other way to find radius

Note: diameter endpoints were used.

D $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(-13-(-3))^2 + (-7-(17))^2}$$

$$= \sqrt{(-10)^2 + (-24)^2}$$

$$= \sqrt{100 + 576}$$

$$= \sqrt{676} = 26$$

diameter

double the radius

$$\frac{26}{2} = \boxed{r = 13}$$