Rational Equations

- equations that contain one or more rational expressions

$$\frac{1}{2}\cdot\frac{x}{1} + \frac{1}{3} = \frac{1}{4}x$$

LCD: 6

* Need a common denominator.

$$\frac{6}{6}\left(\frac{x}{2}\right) + \frac{6}{6}\left(\frac{1}{3}\right) = \frac{x}{4}$$

$$\frac{3x}{6} + \frac{2}{6} = \frac{x}{4}$$

$$\frac{3x+2}{6} = \frac{x}{4}$$

$$4(3x+2) = 6x$$

$$12x + 8 = 6x$$

$$-8 \qquad -8$$

$$12x = 6x - 8$$

$$-6x \qquad -6x$$

$$\frac{6x}{6} = \frac{-8}{6}$$

$$x = -\frac{4}{3}$$

Check $x = -\frac{4}{3}$

$$\frac{1}{2}x + \frac{1}{3} = \frac{1}{4}x$$

$$\frac{1}{2}\left(-\frac{4}{3}\right) + \frac{1}{3} = \frac{1}{4}\left(-\frac{4}{3}\right)$$

$$-\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$

$$-\frac{1}{3} = -\frac{1}{3} \checkmark$$

$x = -\frac{4}{3}$ is a solution

$$\frac{1}{2}\cdot\frac{x}{1} + \frac{1}{3} = \frac{1}{4}x$$

$$\underbrace{\frac{1}{2}\cdot\frac{x}{1} + \frac{1}{3} = \frac{1}{4}x}_{LCD = 12}$$

* Need a common denominator.

$$\frac{12}{12}\left(\frac{x}{2}\right) + \frac{12}{12}\left(\frac{1}{3}\right) = \frac{12}{12}\left(\frac{x}{4}\right)$$

$$\frac{6x}{12} + \frac{4}{12} = \frac{3x}{12}$$

$$\frac{6x+4}{12} = \frac{3x}{12}$$

$$\begin{array}{rcl} 6x+4 & = & 3x \\ -3x & & -3x \\ \hline 3x+4 & = & 0 \\ 3x & = & -4 \end{array}$$

$$\boxed{x = -\frac{4}{3}} \rightarrow \text{Check}$$

$$\underbrace{\frac{1}{2}\cdot\frac{x}{1} + \frac{1}{3} = \frac{1}{4}x}_{\text{LCD: } 12}$$

* Need a common denominator.

$$12\left(\frac{x}{2}\right) + 12\left(\frac{1}{3}\right) = 12\left(\frac{x}{4}\right)$$

$$6x + 4 = 3x$$

$$\vdots$$

$$x = -\frac{4}{3} \longrightarrow$$ check... we did already

* Rational Equations:

1. Find LCD of all fractions
2. Multiply all terms by LCD
3. Solve
4. Check

$$\frac{3}{5} + \frac{1}{x} = \frac{2}{3}$$

LCD: 15x

$$\overset{3}{\cancel{15}}x \cdot \frac{3}{\cancel{5}} + 15\cancel{x} \cdot \frac{1}{\cancel{x}} = \overset{5}{\cancel{15}}x \cdot \frac{2}{\cancel{3}}$$

$$9x + 15 = 10x$$

$$\underline{-9x \qquad\qquad -9x}$$

$$15 = x$$

$\therefore$ x = 15 is the solution

<u>Check</u> $x = 15$

$$\frac{3}{5} + \frac{1}{x} = \frac{2}{3}$$

$$\frac{3}{5} + \frac{1}{15} = \frac{2}{3}$$

$$\frac{\overset{3}{\cancel{15}}}{15} \cdot \frac{3}{\cancel{5}} + \frac{\cancel{15}}{15} \cdot \frac{1}{\cancel{15}} = \frac{\overset{5}{\cancel{15}}}{15} \cdot \frac{2}{\cancel{3}}$$

$$\frac{9+1}{15} = \frac{10}{15}$$

$$\frac{10}{15} = \frac{10}{15} \checkmark$$

* Never move across = when checking.

$$3 - \frac{6w}{w+1} = \frac{6}{w+1}$$

LCD: $w+1$

$$3(w+1) - \frac{6w}{w+1}(w+1) = \frac{6}{w+1}(w+1)$$

$$3w+3 - 6w = 6$$

$$-3w + 3 = 6$$

$$\underline{\quad -3 \quad -3}$$

$$\frac{-3w}{-3} = \frac{3}{-3}$$

$$w = -1$$

Check $w=-1$

$$3 - \frac{6(-1)}{(-1)+1} = \frac{6}{(-1)+1}$$

$$3 - \frac{-6}{0} = \frac{6}{0}$$

undefined fractions

$\rightarrow w=-1$ not a solution

$\therefore$ No solutions

$$3 - \frac{6w}{w+1} = \frac{6}{w+1}$$

$$w+1 \neq 0$$

$$\underline{\quad -1 \quad -1}$$

$$w \neq -1$$

$w=-1$ is an extraneous solution

We can calculate it.

It doesn't really work

$\rightarrow$ if $w=-1$ is calculated to be a solution, reject $w=-1$.

$$\frac{36}{p^2-9} = \frac{2p}{p+3} - 1$$

$$\frac{36}{(p+3)(p-3)} = \frac{2p}{p+3} - 1$$

LCD: $(p+3)(p-3)$

$(p+3)(p-3) \neq 0$

$p+3 \neq 0 \qquad p-3 \neq 0$

$p \neq -3 \qquad p \neq 3$

$\rightarrow$ if $p = -3$ or 3, reject

$$\cancel{(p+3)}\cancel{(p-3)}\frac{36}{\cancel{(p+3)}\cancel{(p-3)}} = \cancel{(p+3)}(p-3)\frac{2p}{\cancel{p+3}} - 1(p+3)(p-3)$$

$$36 = 2p(p-3) - (p+3)(p-3)$$

$$36 = 2p^2 - 6p - (p^2-9)$$

$$36 = 2p^2 - 6p - p^2 + 9$$

$$36 = p^2 - 6p + 9$$

$$\underline{-36 \qquad\qquad -36}$$

$$0 = p^2 - 6p - 27$$

$*a=1$

$-27 = \underline{-9} \cdot \underline{3}$

$-6 = \underline{-9} + \underline{3}$

$$0 = (p-9)(p+3)$$

$\therefore p = 9$ is solution

$p-9=0 \quad$ or $\quad p+3=0$

$\boxed{p=9} \quad$ or $\quad p=-3$ rejected

$* \ LCD \neq 0$

LCD: x^2

$x^2 \neq 0$

$x \neq 0$

$$1 + \frac{3}{x} = \frac{28}{x^2}$$

$$x^2 \cdot 1 + x^2 \cdot \frac{3}{x} = x^2 \cdot \frac{28}{x^2}$$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$x + 7 = 0$ or $x - 4 = 0$

$-7 \ -7 \qquad +4 \ +4$

$\boxed{x = -7} \qquad \boxed{x = 4}$

$-7 \neq 0 \qquad 4 \neq 0$

↳ $\therefore$ $x = -7$ and $x = 4$ are both solutions.

$$x \in \{-7, 4\}$$