$3^3 = 3 \cdot 3 \cdot 3$

$4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

a = base

n = exponent

Properties of exponents

Property

multiplication $b^m \cdot b^n = b^{m+n}$

Examples

$3^2 \cdot 3^3$

$= (3 \cdot 3)(3 \cdot 3 \cdot 3)$

$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= 3^5$

$3^2 \cdot 3^3$

$= 3^{2+3}$

$= 3^5$

Property

Division $\frac{b^m}{b^n} = b^{m-n}$, $b \neq 0$

Example

$\frac{5^6}{5^2} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot \cancel{5} \cdot \cancel{5}}{\cancel{5} \cdot \cancel{5}} = 5 \cdot 5 \cdot 5 \cdot 5 = 5^4$

$= 5^{6-2} = 5^4$

Property

Power of power

$$(b^m)^n = b^{m \cdot n}$$

e.g.

$$(6^2)^4 = 6^2 \cdot 6^2 \cdot 6^2 \cdot 6^2$$
$$= 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$$
$$= 6^8$$

$$(6^2)^4 = 6^{2 \cdot 4} = 6^8$$

$$(6^4)^2 = 6^4 \cdot 6^4$$
$$= (6 \cdot 6 \cdot 6 \cdot 6)(6 \cdot 6 \cdot 6 \cdot 6)$$
$$= 6^8$$

$$(6^4)^2 = 6^{4 \cdot 2} = 6^8$$

Property

Power of product

$$(ab)^n = a^n \cdot b^n$$

e.g.

$$(2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$$
$$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$$
$$= 2^3 \cdot 3^3$$

Property
Power of Quotient

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad b \neq 0$$

e.g.

$$\left(\frac{6}{5}\right)^4 = \left(\frac{6}{5}\right)\left(\frac{6}{5}\right)\left(\frac{6}{5}\right)\left(\frac{6}{5}\right)$$

$$= \frac{6 \cdot 6 \cdot 6 \cdot 6}{5 \cdot 5 \cdot 5 \cdot 5}$$

$$= \frac{6^4}{5^4}$$

$b^0 = ?$

$b^3 = b \cdot b \cdot b$

$b^2 = b \cdot b = \frac{b^3}{b}$

$b^1 = b = \frac{b^2}{b}$

$b^0 = \frac{b^1}{b} = 1$

$b^{-1} = \frac{b^0}{b} = \frac{1}{b} = \frac{1}{b^1}$

$b^{-2} = \frac{b^{-1}}{b} = \frac{\frac{1}{b}}{b} = \frac{1}{b} \div b = \frac{1}{b} \cdot \frac{1}{b} = \frac{1}{b^2}$

Zero power

$$b^0 = 1 \qquad b \neq 0$$

Negative Exponents

$$b^{-n} = \frac{1}{b^n} \qquad b \neq 0$$

* cannot divide by zero.

* $0^0 \neq 1$

$0^n = 0$
$b^0 = 1$
→ contradiction

0^0 is an indeterminate form

$$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = 1 \div \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^n$$

$$= 1 \div \frac{a^n}{b^n}$$

$$= 1 \cdot \frac{b^n}{a^n}$$

$$= \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$$

e.g. $\left(\frac{17}{6}\right)^{-3} = \left(\frac{6}{17}\right)^3 = \frac{6^3}{17^3}$

$$= \left(\frac{6}{17}\right)\left(\frac{6}{17}\right)\left(\frac{6}{17}\right) = \frac{6^3}{17^3} = \left(\frac{6}{17}\right)^3$$

Practice

$(-2)^4 = (-2)(-2)(-2)(-2) = (-1)^4(2)^4 = 1 \cdot 16 = 16$

$-2^4 = -1 \cdot 2^4 = -1\,(2)(2)(2)(2) = -1 \cdot 16 = -16$

$-2^{-4} = -1 \cdot 2^{-4} = -1 \cdot \frac{1}{2^4} = -1 \cdot \frac{1}{16} = \frac{1}{-16} = \frac{-1}{16} = -\frac{1}{16}$

$(-3)^3 = (-1 \cdot 3)^3 = (-1)^3(3)^3 = -1 \cdot 27 = -27$

$-3^3 = -1 \cdot 3^3 = -1 \cdot 27 = -27$

$(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27} = -\frac{1}{27}$

* odd number of negatives
  → negative value

* even number of negatives
  → positive value

$(-7x)^0 = 1$ by 0 power property

$= (-7)^0 (x)^0 = 1 \cdot 1 = 1$

by product property

$-7x^0 = -7(1) = -7$

$x^3 x^5 x^{-2} = x^{3+5+(-2)} = x^6$

$\frac{y^7}{y^4} = y^{7-4} = y^3$ $\qquad \frac{y^7}{y^4} = \frac{y^3}{1}$

$\frac{y^4}{y^7} = y^{4-7} = y^{-3} = \frac{1}{y^3}$ $\qquad \frac{y^4}{y^7} = \frac{1}{y^3}$

$(b^2)^{-5} = b^{(2)(-5)} = b^{-10} = \frac{1}{b^{10}}$

$$\left(\frac{1}{5}\right)^{-3} - (2)^{-2} + 3^0$$

$$\left(\frac{5}{1}\right)^3 - \frac{1}{2^2} + 1 =$$

$$125 - \frac{1}{4} + 1 =$$

$$\left(\frac{4}{4}\right)126 - \frac{1}{4} =$$

$$\frac{504}{4} - \frac{1}{4} = \frac{503}{4}$$

$$\frac{(2a^7b^{-4})^3}{(4a^3b^{-2})^2} = \frac{2^3 a^{7\cdot 3} b^{-4\cdot 3}}{4^2 a^{3\cdot 2} b^{-2\cdot 2}}$$

$$= \frac{8a^{21}b^{-12}}{16a^6b^{-4}}$$

$$= \frac{8}{16}\frac{1}{2} a^{21-6} b^{-12-(-4)}$$

$$= \frac{1}{2} a^{15} b^{-8}$$

$$= \frac{1}{2}\left(\frac{a^{15}}{1}\right)\left(\frac{1}{b^8}\right)$$

$$= \frac{a^{15}}{2b^8}$$

$$\left(\frac{8x^2}{3x^5y^2}\right)^{-2} = \left(\frac{8}{3y^2}x^{2-5}\right)^{-2}$$

$$= \left(\frac{8}{3y^2}x^{-3}\right)^{-2}$$

$$\left(\frac{8}{3x^{5-2}y^2}\right)^{-2} \rightarrow = \left(\frac{8}{3x^3y^2}\right)^{-2}$$

$$= \left(\frac{3x^3y^2}{8}\right)^{2}$$

* negative exponent flip

$$= \frac{3^2x^{3\cdot 2}y^{2\cdot 2}}{8^2}$$

$$= \frac{9x^6y^4}{64}$$