A linear equation has an infinite number of solutions that form a line in the Cartesian plane.

A system of linear equations is two or more in the near equations

Consider:

$$3x+2y=-8$$

$$y=2x-4$$

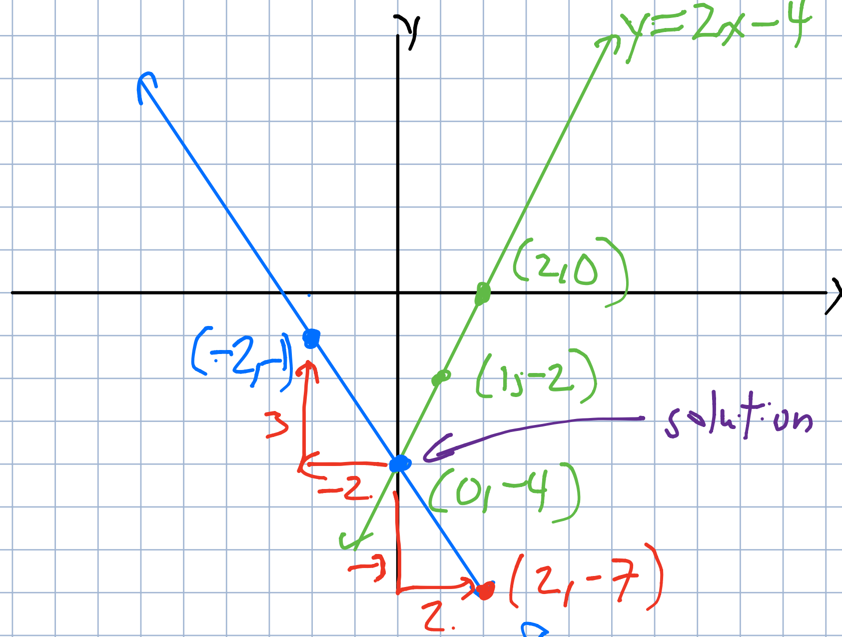
$$y=2x-4$$

$$y=mx+b$$

$(0,b)$ is y int

$(0,-4)$

slope $m=\frac{2}{1}$ $\frac{\uparrow 2}{\rightarrow 1}$



$$3x+2y=-8$$

$$\underline{-3x \qquad\qquad -3x}$$

$$\frac{2y}{2}=\frac{-3x-8}{2}$$

$$y=-\frac{3}{2}x-4$$

y-int $(0,-4)$

$$m=-\frac{3}{2}$$

* solution: $(0,-4)$ is the point of intersection

Solving the system algebraically.

$$3x+2y=-8 \rightarrow 3x+2(2x-4)=-8$$

$$y=2x-4$$

$$3x+4x-8=-8$$
$$\quad\quad +8 \quad +8$$
$$\frac{7x}{7}=\frac{0}{7}$$

$$x=0$$

Need to solve for y, Let $x=0$

It doesn't matter which equation we use.

$$y=2x-4$$
$$y=2(0)-4$$
$$y=0-4$$
$$y=-4$$

Solution
$$(x,y)=(0,-4)$$

$3x+2y=-8 \rightarrow$ from earlier

$y=-\frac{3}{2}x-4$

$y=2x-4$

*If $a=b$ and $b=c$, then $a=c$.

*Transitive Property of equality

$\rightarrow$ Since both expressions equal y

$$2x\cancel{-4} = -\frac{3}{2}x\cancel{-4}$$

$$\cancel{+4} \qquad\qquad \cancel{+4}$$

$$2x = \cancel{-\frac{3}{2}x}$$

$$+\frac{3}{2}x \qquad \cancel{+\frac{3}{2}x}$$

$$2\left(\frac{2}{2}\right)x+\frac{3}{2}x=0$$

$$\frac{4}{2}x+\frac{3}{2}x=0$$

$$\frac{7}{2}x=0$$

$$x=0$$

by Zero Product Property

Method of substitution

Isolate one variable from one equation.
Substitute quantity found in step one into other equation.
Solve the resulting equation.
Substitute resulting value into original equations to find other variable
Check.

Consider:

$$x + y = 16$$
$$x - y = 4$$

$$\frac{2x}{2} = \frac{20}{2}$$
$$x = 10$$

*Observe:
Two equations in standard form
$$Ax + By = C$$

$+y$, $-y$

when we add both equations one variable is eliminated

Solve for y, $x = 10$

$$x + y = 16$$
$$(10) + y = 16$$
$$-10 \qquad -10$$
$$y = 6$$

or

$$x - y = 4$$
$$(10) - y = 4$$
$$-10 \qquad -10$$
$$\frac{-y}{-1} = \frac{-6}{-1}$$
$$y = 6$$

Solution $(x, y) = (10; 6)$

$$3x - 2y = -7$$
$$6x + y = 6$$

$$3x - 2y = -7$$
$$2(6x + y) = 2(6)$$

multiplied 2nd equation by 2 in order to cancel y

$$3x - 2y = -7$$
$$12x + 2y = 12$$

$$\frac{15x}{15} = \frac{5}{15} = \frac{(\cancel{5})(1)}{(\cancel{5})(3)}$$

$$x = \frac{1}{3}$$

Find y, $x = \frac{1}{3}$

$$6x + y = 6$$
$$\frac{6}{1}\left(\frac{1}{3}\right) + y = 6$$
$$\cancel{2} + y = 6$$
$$-\cancel{2} \qquad -2$$
$$y = 4$$

Solution

$$(x, y) = \left(\frac{1}{3}, 4\right)$$

* Observations

in standard form

one positive coefficient for y

" negative " " "

$2x + 3y = 7$

$x + y = 3$

$2x + 3y = 7$

$-2x - 2y = -6$

$y = 1$

$x + y = 3$

$x + (1) = 3$

$\quad -1 \quad -1$

$x = 2$

* Observations

all addition

multiply by -2 to get $+2x$ and $-2x$

$(x, y) = (2, 1)$

A $3x+2y=4$

B $4x+3y=7$

4 A $12x+8y=16$

-3 B $-12x-9y=-21$

$$-y=-5$$

$$y=5$$

$$4x+3y=7$$

$$4x+3(5)=7$$

$$4x+15=7$$

$$-15 \quad -15$$

$$\frac{4x}{4}=\frac{-8}{4}$$

$$x=-2$$

*Observation

* no coefficients are multiples of other coefficients

→ must multiply both

* chosen to eliminate x

$$(x,y)=(-2,5)$$

Addition/Elimination Method

Both equations standard form
Multiply one or both equations by 90 constant to create additive inverse coefficients for one variable
Add equations from previous steps to eliminate one variable
Subs to answer from previous step into one of the original equations to solve for the other variable
Check.

$$3x+2y=4 \longrightarrow \quad 3x+2y=4$$
$$-3x \qquad -3x$$
$$2y = 4-3x$$
$$y = \frac{4-3x}{2}$$
$$y = 2-\frac{3}{2}x$$

$$4x+3y=7$$

$$4x+3y=7$$
$$4x+3\left(2-\frac{3}{2}x\right)=7$$
$$4x+6-\frac{9}{2}x=7$$
$$\left(\frac{2}{2}\right)4x-\frac{9}{2}x+6=7$$
$$-6 \quad -6$$
$$\left(\frac{8}{2}-\frac{9}{2}\right)x = 1$$
$$\frac{-\frac{1}{2}x}{-\frac{1}{2}} = \frac{1}{-\frac{1}{2}}$$
$$x=-2$$
$$\vdots$$
$$y=5$$

$$(x,y)=(-2,5)$$