

Find the line passing through
 $(-2, 6)$ and $(-8, 3)$.

$x_1 \ y_1$ $x_2 \ y_2$

1. Need slope.

2. Types of Linear Equations

standard

$$Ax + By = C$$

slope-intercept

$$y = mx + b$$

point-slope

$$y = m(x - x_1) + y_1$$

$$\begin{aligned} 1. \ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(3) - (6)}{(-8) - (-2)} \end{aligned}$$

$$= \frac{-3}{-8 + 2}$$

$$= \frac{-3}{-6}$$

$$= \frac{1}{2}$$

2. Find line

Point slope $m = \frac{1}{2}$ $(-2, 6)$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{2}(x - (-2)) + (6)$$

$$y = \frac{1}{2}(x + 2) + 6$$

$m = \frac{1}{2}$ $(-8, 3)$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{2}(x - (-8)) + (3)$$

Slope Intercept

Option 1. Convert from point-slope to slope-intercept

$$a.) y = \frac{1}{2}(x+2) + 6$$

$$y = \frac{1}{2}x + \frac{2}{2} + 6$$

$$y = \frac{1}{2}x + 1 + 6$$

$$y = \frac{1}{2}x + 7$$

$$b.) y = \frac{1}{2}(x - (-8)) + 3$$

$$y = \frac{1}{2}(x + 8) + 3$$

$$y = \frac{1}{2}x + \frac{8}{2} + 3$$

$$y = \frac{1}{2}x + 4 + 3$$

$$y = \frac{1}{2}x + 7$$

Option 2. Find b.

$$m = \frac{1}{2} \quad y = mx + b$$

Point on line

$$(-8, 3) \quad (3) = \left(\frac{1}{2}\right)(-8) + b$$

$$3 = -4 + b$$

$$+4 \quad +4$$

$$7 = b$$

Substitute m and b

$$y = \frac{1}{2}x + 7$$

* Should result in same equation w/ $(-2, 6)$

Convert Standard form $\rightarrow y = mx + b$

$$Ax + By = C \rightarrow y = mx + b$$

eg. $3x + 4y = 4$
 $\frac{-3x}{-3x} \quad \frac{-3x}{-3x}$

$$\frac{4y}{4} = \frac{4 - 3x}{4}$$

$$y = \frac{4 - 3x}{4}$$

$$y = \frac{4}{4} - \frac{3x}{4}$$

$$y = 1 - \frac{3}{4}x$$

$$y = +1 + (-\frac{3}{4}x)$$

Objective: solve for y
in terms of x .

$$\frac{1}{4}(4 - 3x)$$

$$y = -\frac{3}{4}x + 1$$

$$\frac{3x + 4y = 4}{-3x \quad -3x}$$

$$\frac{4y}{4} = \frac{-3x + 4}{4}$$

$$y = -\frac{3}{4}x + 1$$

Graphing Lines

1. slope-intercept form

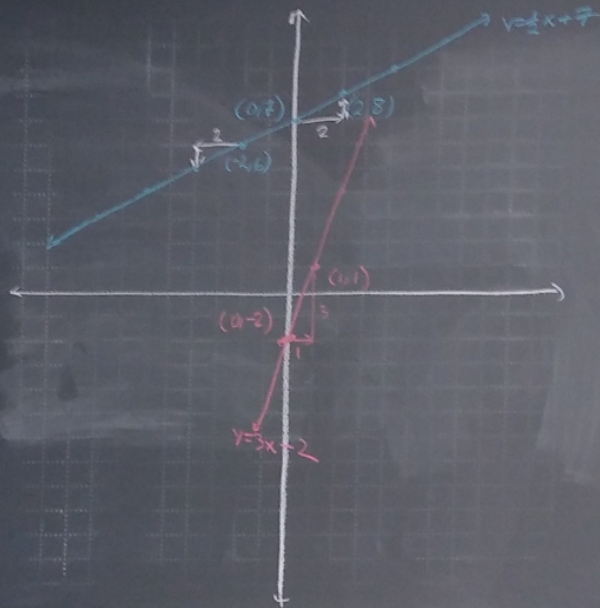
$$y = mx + b$$

$$y = \frac{1}{2}x + 7$$

$$m = \frac{1}{2}$$

$(0, 7)$

$\uparrow 1$ $2 \rightarrow$



$$y = 3x - 2$$

$$m = \frac{3}{1}$$

$(0, -2)$

$\rightarrow 1$ $\uparrow 3$

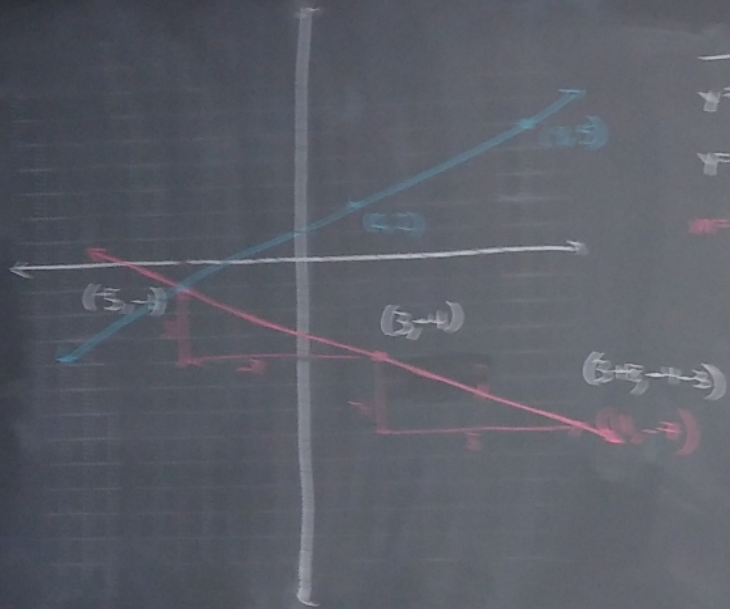
Graphing Lines

$$y = \frac{1}{2}(x-1) + 5$$

(1, 5)

$\frac{1}{2}$

$\uparrow + 2$
 $\downarrow - 1$
 $\uparrow + 1$
 $\downarrow - 2$



Graphing in point-slope form

$$y = m(x - x_1) + y_1$$

$$y = \frac{3}{2}(x - 3) - 4$$

$$m = \frac{3}{2}$$

$$(x_1, y_1) = (3, -4)$$

$\downarrow 3 \rightarrow 8$

Graphing in standard form

$$2x - 5y = 20$$

* Observe that 20 is divisible by 2 and 5.

→ Look for x and y-intercepts

y-intercept: y-value when $x=0$

$$\begin{aligned} \rightarrow 2(0) - 5y &= 20 \\ 0 - 5y &= 20 & (0, -4) \\ \frac{-5y}{-5} &= \frac{20}{-5} \\ y &= -4 \end{aligned}$$

x-intercept: x when $y=0$

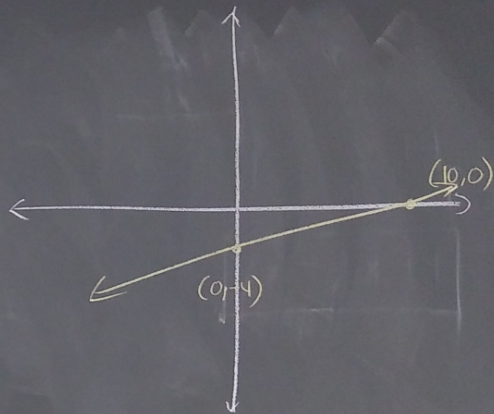
$$2x - 5(0) = 20$$

$$2x - 0 = 20$$

$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

$(10, 0)$



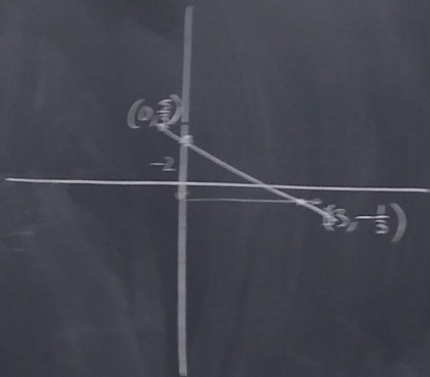
$$2x + 3y = 5$$

$$\begin{array}{r} -2x \quad -2x \\ \hline \end{array}$$

$$\frac{3y}{3} = \frac{-2x + 5}{3}$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

Graph this



parallel lines

lines if extended infinitely will never touch.

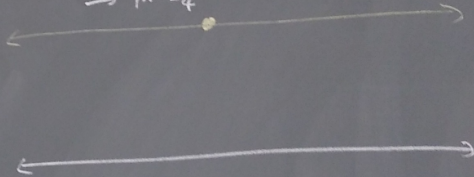
Playfair's Axiom

Given a line and a point outside the line, then there exists only one line that can pass through the point that is parallel to the original line.

lines are parallel if they have the same slope.

$$m_1 = m_2 \rightarrow l_1 \parallel l_2$$

$$\begin{array}{r|l} 3x + 4y = 4 & 6x + 8y = 5 \\ -3x & -6x \\ \hline 4y = -3x + 4 & 8y = -6x + 5 \\ \frac{4y}{4} = \frac{-3x + 4}{4} & \frac{8y}{8} = \frac{-6x + 5}{8} \\ \rightarrow m = -\frac{3}{4} & m = -\frac{6}{8} = -\frac{3}{4} \end{array}$$



$$y = -\frac{5}{6}(x+8) + 1, \quad y = -\frac{5}{6}x + 5 \quad (m = -\frac{5}{6})$$

e.g. $y = 5$ and $y = 7$ (both horizontal, both $m = 0$)

$$y = 2x + 3 \quad y = 2x + 5 \quad (\text{slope } m = 2)$$

$$x = \frac{1}{2} \quad x = \frac{5}{3} \quad (\text{both vertical})$$

perpendicular lines

are lines that intersect at 90° angle

$$m_1 = -\frac{1}{m_2}$$

$$m = \frac{\Delta y}{\Delta x} \rightarrow m_{\perp} = -\frac{\Delta x}{\Delta y}$$

\perp slopes are negative reciprocals

Find lines that are parallel and perpendicular to $y = -\frac{5}{6}(x+8) + 1$ that passes through $(-3, 1)$

parallel

$$\rightarrow m = -\frac{5}{6}$$

$(-3, 1)$

point-slope: $y = m(x - x_1) + y_1$

$$y = -\frac{5}{6}(x - (-3)) + (1)$$

$$y = -\frac{5}{6}(x + 3) + 1$$

perpendicular

$$\rightarrow m = -\left(-\frac{6}{5}\right) = \frac{6}{5}$$

$(-3, 1)$

$$y = \frac{6}{5}(x - (-3)) + (1)$$

$$y = \frac{6}{5}(x + 3) + 1$$