Consider the equation

$$2x + 5y = 20$$

$$\begin{array}{rr} -2x & -2x \\ \hline \end{array}$$

$$5y = -2x + 20$$

$$\frac{5y}{5} = \frac{-2x}{5} + \frac{20}{5}$$

$$y = -\frac{2}{5}x + 4$$

Let $x = 0$

$$2(0) + 5y = 20$$

$$\frac{5y}{5} = \frac{20}{5}$$

$$y = 4$$

Let $y = 0$

$$2x + 5y = 20$$

$$2x + 5(0) = 20$$

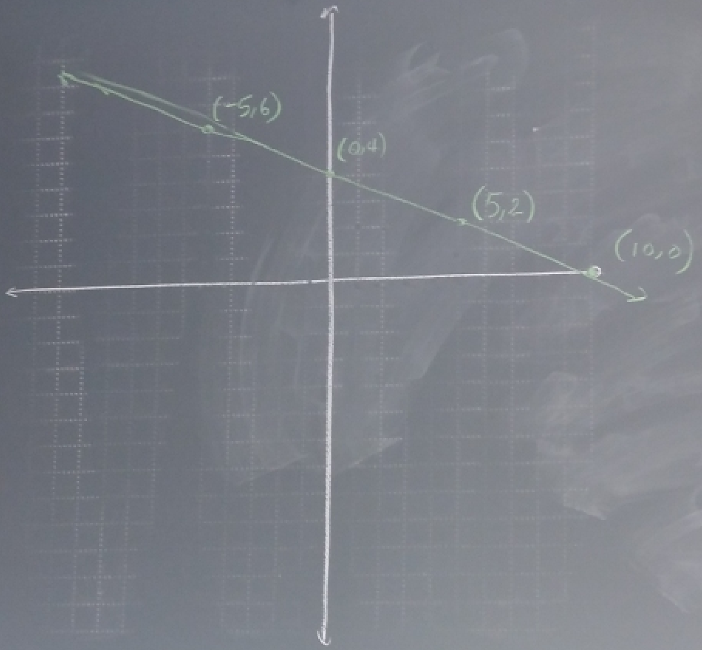
$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

Solution

$$(x, y) = (0, 4)$$

$$(x, y) = (10, 0)$$


(-5,6)
(0,4)
(5,2)
(10,0)

Check (5, 2)

$2x + 5y = 20$

$2(5) + 5(2) = 20$

$10 + 10 = 20$

$20 = 20$ ✓

→ We can confirm that $(5,2)$ is a solution to $2x + 5y = 20$

Check $(-5, 6)$

$2(-5) + 5(6) = 20$

$-10 + 30 = 20$

$20 = 20$ ✓

→ $(-5, 6)$ is a solution.

∴ Any point on a line is a solution to a linear equation.

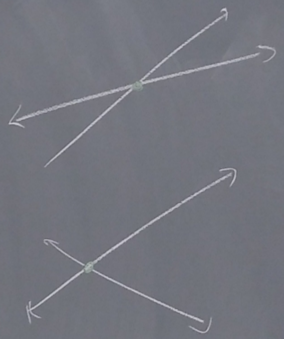
Working with 2 equations

$3x + 2y = -8$ ← standard form
$Ax + By = C$

$y = 2x - 4$ ← slope intercept form
$y = mx + b$

When graphing, $(0,-4)$ satisfies both equations

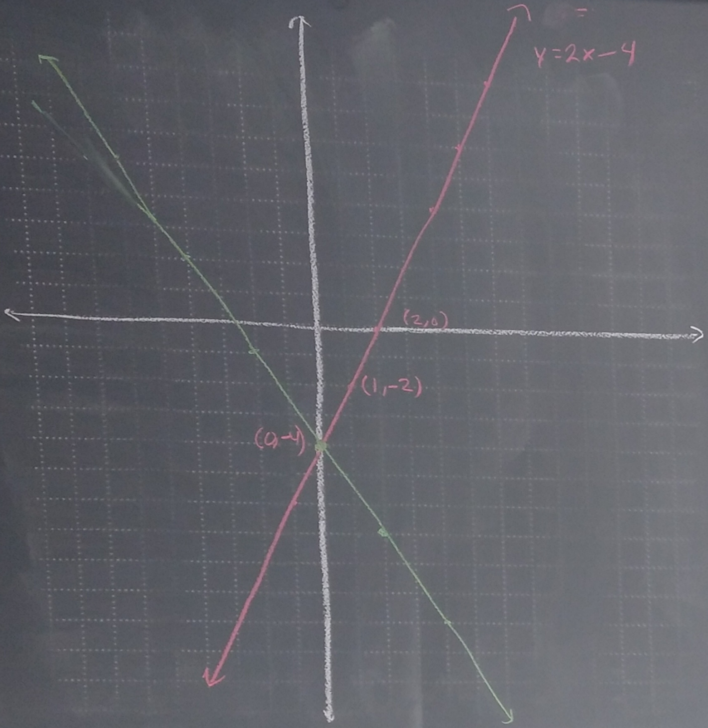
→ $(0,-4)$ is the solution to the system of equations.



if two different slopes
→ expect one solution
↑
point of intersection

Parallel lines would have no solution (no intersection points)

Two of the same line has infinite solutions.



$y=2x-4$

y-int $(0,-4)$ slope $\frac{2}{1}$

$3x+2y=-8$ → convert

Let $x=0$

$3(0)+2y=-8$

$\frac{2y}{2}=\frac{-8}{2}$

$y=-4$

y-int → $(0,-4)$

$3x+2y=-8$

$-3x \qquad -3x$

$\frac{2y}{2}=\frac{-3x-8}{2}$

$y=-\frac{3}{2}x-\frac{8}{2}$

$y=-\frac{3}{2}x-4$

slope: $-\frac{3}{2}$, $(0,-4)$

Working with 2 equations

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$3x + 2y = -8$

① $y = 2x - 4$

1. Isolate one variable from one equation
2. Substitute result from step 1 into the other equation.
3. Solve resulting equation
4. Use solution to solve for other variable.

②

$$3x + 2y = -8 \rightarrow 3x + 2(2x-4) = -8$$

$$3x + 4x - 8 = -8$$

$$+8 \quad +8$$

$$3x + 4x = 0$$

$$7x = 0$$

$$\frac{7x}{7} = \frac{0}{7}$$

③ $x = 0$

4

Let $x=0$

$y=2x-4$

$y=2(0)-4$

$y=0-4$

$y=-4$

$(0,-4)$ is solution

$3x+2y=-8$

$3(0)+2y=-8$

$0+2y=-8$

$2y=-8$

$\frac{2y}{2}=\frac{-8}{2}$

$y=-4$

$(0,-4)$ is solution

Use substitution method

$x + y = 16$

$x - y = 4$

① Solve for y.

using 1st equation, $x + y = 16$

$$\begin{aligned} x + y &= 16 \\ -x \quad & \quad -x \\ \hline y &= -x + 16 \end{aligned}$$

② Use y in other equation. $x - y = 4$

③ Solve resulting equation $x - (-x + 16) = 4 \rightarrow x = 10$

$x + x - 16 = 4$

$2x - 16 = 4$

$\quad +16 \quad +16$

$\dfrac{2x}{2} = \dfrac{20}{2}$

④ Solve for y using $x = 10$

Chose $x + y = 16$

$$\begin{aligned} (10) + y &= 16 \\ -10 \quad & \quad -10 \\ \hline y &= 6 \end{aligned}$$

$\boxed{\rightarrow (x, y) = (10, 6)}$

$$x+y=16$$
$$x-y=4$$

① Solve for x.

2nd $x-y=4$
$$\begin{array}{r} x-y=4 \\ +y\ \ +y \\ \hline x\ \ \ =4+y \end{array}$$

② Substitute into 1st Eqn.

$$x+y=16$$
$$(4+y)+y=16$$

③ Solve

$$\begin{array}{r} 4+2y=16 \\ -4\ \ \ \ \ -4 \\ \hline 2y=12 \\ y=6 \end{array}$$

④ Solve for x using $y=6$

$$x+y=16$$
$$\begin{array}{r} x+(6)=16 \\ -6\ \ \ -6 \\ \hline x\ \ \ =10 \end{array}$$

$\longrightarrow (x,y)=(10,6)$

$-\frac{3}{2}$

4)

$x+y=16$

$\longrightarrow \quad y=-x+16$

What happens if we substitute back into $x+y=16$?

$x+y=16$

$x+(-x+16)=16$

$x-x+16=16$

$16=16 \longrightarrow$ infinite solutions

Note: substituting line back into itself

Use substitution method (if you can solve for a variable quickly.)

$$x + y = 16$$
$$x - y = 4$$
$$2x = 20$$
$$\frac{2x}{2} = \frac{20}{2}$$
$$x = 10$$

Observe "signs" for y are different

Add the two equations together.

→ y cancelled, left with x.

→ Note. we have done this before so we know (10,6)

A $3x - 2y = -7$

B $6x + y = 6$

A $3x - 2y = -7$

2B $12x + 2y = 12$

$$15x = 5$$
$$\frac{15x}{15} = \frac{\cancel{5}\ 1\cdot\cancel{5}}{\cancel{15}\ 3\cdot\cancel{5}}$$
$$x = \frac{1}{3}$$

Observe cannot easily cancel one variable.

We've chosen to get rid of y

Multiply 2nd equation

2B $2(6x + y) = 2(6)$

$12x + 2y = 12$

Solve for y using $x = \frac{1}{3}$

Ⓐ $3x - 2y = -7$

$3\left(\frac{1}{3}\right) - 2y = -7$

$1 - 2y = -7$

$-1 \qquad -1$

$-2y = -8$

$\frac{-2y}{-2} = \frac{-8}{-2}$

$y = 4$

$\rightarrow \left(\frac{1}{3}, 4\right)$

Ⓐ $2x + 3y = 7$

Ⓑ $x + y = 3$

Ⓐ $2x + 3y = 7$

-2 Ⓑ $-2x - 2y = -6$

$y = 1$

Ⓑ $x + y = 3$

$x + (1) = 3$

$-1 \quad -1$

$x = 2$

$\boxed{(2,1)}$

Addition / Elimination Method

1. Both equations in standard form
2. Clear Fractions / Decimals
3. Select a variable to **eliminate**.
4. Multiply one or both equations by a non zero number.
   → Make sure + coefficient, − coefficient with same variable in both equations
5. **Add** the equations to eliminate chosen variable.
6. Solve
7. Substitute solution in step 6 to find other variable.

A $3x + 2y = 4$

B $4x + 3y = 7$

-4 A $-12x - 8y = -16$ ← Choose to eliminate x.

3 B $12x + 9y = 21$

$y = 5$

A $3x + 2y = 4$

$3x + 2(5) = 4$

$3x + 10 = 4$

$-10 \quad -10$

$3x = -6$

$x = -2$

$(x, y) = (-2, 5)$