$3^x = 27$

$3^x = 3^3$ ← simple stuff

$x = 3$

$6^x = 215$

$\ln(6^x) = \ln(215)$

$x\ln(6) = \ln(215)$

$\frac{x\ln(6)}{\ln(6)} = \frac{\ln(215)}{\ln(6)}$

$x = \frac{\ln(215)}{\ln(6)}$

$x \approx 2.997$

* We don't observe that 6 and 215 have common factors

$6 = 3 \cdot 2$

3 2

$215 = 5 \cdot 43$

5 43

log or ln both sides

power rule for logarithms

$\log_c(b^n) = n\log_c(b)$

$6^x = 215$

$\log(6^x) = \log(215)$

$x\log(6) = \log(215)$

$x = \frac{\log(215)}{\log(6)}$

$x \approx 2.997$

$6^x = 215 \longleftrightarrow \log_6(215) = x$

$$x = \frac{\ln(215)}{\ln(6)} = \frac{\log(215)}{\log(6)}$$

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)} = \frac{\ln(a)}{\ln(b)}$$

* change of base property

$7^x = 33$

$\ln(7^x) = \ln(33)$

$x\ln(7) = \ln(33)$

$$\frac{x\ln(7)}{\ln(7)} = \frac{\ln(33)}{\ln(7)}$$

$$x = \frac{\ln(33)}{\ln(7)}$$

$\boxed{x \approx 1.797}$

$7^x = 33$

$\log(7^x) = \log(33)$

$x\log(7) = \log(33)$

$$\frac{x\cancel{\log(7)}}{\cancel{\log(7)}} = \frac{\log(33)}{\log(7)}$$

$$x = \frac{\log(33)}{\log(7)}$$

$\boxed{x \approx 1.797}$

$$e^x = 52.2049$$

$$\ln(e^x) = \ln(52.2049)$$

$$x \ln(e) = \ln(52.2049)$$

$$x(1) = \ln(52.2049)$$

$$x = \ln(52.2049)$$

$$\boxed{x \approx 3.955}$$

* $\ln(e) = 1$

* $\ln(e) = \log_e(e) = 1$

$$e^x = 52.2049$$

$$\log(e^x) = \log(52.2049)$$

$$x \log(e) = \log(52.2049)$$

$$\frac{x \log(e)}{\log(e)} = \frac{\log(52.2049)}{\log(e)}$$

$$x = \frac{\log(52.2049)}{\log(e)}$$

$$\boxed{x \approx 3.955}$$

* natural log looks easier

if $e^x = \ldots$

use Ln

$10^x = 0.00477$

$\log(10^x) = \log(0.00477)$

$x \log_{10}(10) = \log_{10}(0.00477)$

$x\,(1) = \log(0.00477)$

$x = \log(0.00477)$

$\boxed{x \approx -2.321}$

* if $10^x \rightarrow$ use log

$$20 \cdot 1{,}2^x = 37$$

$$\frac{20 \cdot 1{,}2^x}{20} = \frac{37}{20}$$

$$1{,}2^x = \frac{37}{20}$$

$$\ln\left(1{,}2^x\right) = \ln\left(\frac{37}{20}\right)$$

$$x \ln(1{,}2) = \ln(1{,}85)$$

$$\frac{x \ln(1.2)}{\ln(1.2)} = \frac{\ln(1{,}85)}{\ln(1.2)}$$

$$x = \frac{\ln(1{,}85)}{\ln(1.2)}$$

$$x \approx 3{,}374\ldots$$

$$5^{x+3} = 9^{x+1}$$

*Solve for x

$$\ln\left(5^{x+3}\right) = \ln\left(9^{x+1}\right)$$

$$(x+3)\cdot\ln(5) = (x+1)\cdot\ln(9)$$

$$x\ln(5) + \cancel{3\ln(5)} = \cancel{x\ln(9)} + \ln(9)$$

$$-x\ln(9) - \cancel{3\ln(5)} \quad - \cancel{x\ln(9)} \quad -3\ln(5)$$

*We need x on one side of equations

$$x\ln(5) - x\ln(9) = \ln(9) - 3\ln(5)$$

$$x\left(\ln(5) - \ln(9)\right) = \ln(9) - 3\ln(5)$$

$$\frac{x\cancel{\left(\ln(5) - \ln(9)\right)}}{\cancel{\left(\ln(5) - \ln(9)\right)}} = \frac{\ln(9) - 3\ln(5)}{\ln(5) - \ln(9)}$$

$$x = \frac{\ln(9) - 3\ln(5)}{\ln(5) - \ln(9)} \approx 4.476$$

$$x = \frac{\ln(9) - \ln(125)}{\ln(5) - \ln(9)}$$

$$x = \frac{\ln\left(\frac{9}{125}\right)}{\ln\left(\frac{5}{9}\right)}$$

Exercise 2. A bacterial culture of 20g has been cultivated, which naturally increases at a rate of 3.5% per week.

(a) What will be the weight of the culture after 6 weeks?

$$A = P(1+r)^t$$
$$A = (20)(1+.035)^6$$
$$= 20(1.035)^6$$
$$= 24.585\text{g}$$

$A = ?$
$P = 20\text{g}$
$r = .035$
$t = 6$

(b) How long will it take until the culture has doubled in weight?

$$A = P(1+r)^t$$
$$40 = 20 \cdot (1+.035)^t$$
$$\frac{40}{20} = \frac{\cancel{20} \cdot (1.035)^t}{\cancel{20}}$$
$$2 = (1.035)^t$$
$$\ln(2) = \ln(1.035^t)$$
$$\ln(2) = t \cdot \ln(1.035)$$
$$\frac{\ln(2)}{\ln(1.035)} = \frac{t\,\cancel{\ln(1.035)}}{\cancel{\ln(1.035)}}$$
$$20.15 \text{ weeks} \approx t$$

$t \approx 20$ weeks 1 day

$A = 40$
$P = 20$
$r = .035$
$t = ?$

Exercise 3. A radioactive substance decays with a half-life of 4 hours. How long will it take until 34mg will have decayed to 10mg?

half-life

$$A = P(.5)^{\frac{t}{h}}$$

h = half life

A = 10

P = 34mg

t = ?

h = 4

$$\frac{(10)}{34} = \frac{\cancel{(34)} \cdot (.5)^{\frac{t}{(4)}}}{\cancel{34}}$$

$$\frac{5}{17} = .5^{\frac{t}{4}}$$

$$\ln\left(\frac{5}{17}\right) = \ln\left(.5^{\frac{t}{4}}\right)$$

* power rule for exponents

$$\ln\left(\frac{5}{17}\right) = \frac{t}{4} \cdot \ln(.5)$$

$$\ln\left(\frac{5}{17}\right) = t \cdot \frac{\ln(.5)}{4}$$

$$\frac{4}{\ln(.5)} \ln\left(\frac{5}{17}\right) = t \cdot \frac{\ln(.5)}{4} \cdot \frac{4}{\ln(.5)}$$

$$* \quad \frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ab} = 1$$

$$\frac{4\ln\left(\frac{5}{17}\right)}{\ln(.5)} = t$$

$$\boxed{7.062 \text{ hours} \approx t}$$

Exercise 4. A piece of wood has lost 12% of its carbon-14. How old is the wood?

$$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}} \leftarrow \text{half-life}$$

$A = 100\% - 12\%$
88%
$= .88P$

$P = P$

$t =$

$h = 5730$ years
(half-life C-14)

$$(.88P) = (P)\left(\frac{1}{2}\right)^{\left(\frac{t}{5730}\right)}$$

$$\frac{.88\cancel{P}}{\cancel{P}} = \frac{\cancel{P}\left(\frac{1}{2}\right)^{\frac{t}{5730}}}{\cancel{P}}$$

note: $P \neq 0$

$$.88 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$\ln(.88) = \ln\left(\left(\frac{1}{2}\right)^{\frac{t}{5730}}\right)$$

$$\ln(.88) = \frac{t \ln\left(\frac{1}{2}\right)}{5730}$$

$$\frac{5730 \ln(.88)}{\ln\left(\frac{1}{2}\right)} = \frac{t \ln\left(\frac{1}{2}\right)}{5730} \cdot \frac{5730}{\ln\left(\frac{1}{2}\right)}$$

$$\boxed{\begin{aligned} * & \ln\left(\tfrac{1}{2}\right) \\ &= \ln\left(2^{-1}\right) \\ &= -\ln(2) \end{aligned}}$$

$$\frac{5730 \ln(.88)}{-\ln(2)} = t$$

$$\boxed{t \approx 1057 \text{ years}}$$

(1 point) **CUNY/CityTech/CollegeAlgebra_Trig/ExponentialEquations-Calc/population-word.pg**

The population of Egypt can be modeled by the function

original population 114 million

$$P(t) = 114(1 + 0.018)^t$$

where $P(t)$ measures the population in millions and t represents the number of years since 2000.

1. Using this model, what was the population of Egypt in 2007? $t=7$
2. Predict the population of Egypt in 2023. $t=23$
3. If this growth rate continues, in what year will the population of Egypt reach 2 billion people?

Hint:

$$P(t) = 114 \cdot (1.018)^t$$

more decimal 6 →

1. $P(7) = 114(1.018)^7 \approx 129.1633491$ millions

* To get population, multiply by 1 000 000

$\approx 129\ 163\ 349.1$ people

$\approx 129\ 163\ 349$ people

2. $P(23) = 114(1.018)^{23} \approx 171.8318814$ million

→ 171 831 881 people

3. $t = ?$

$$P(t) = 114(1.018)^t$$

in millions

$$P(t) = 2 \text{ billion}$$
$$= 2000 \text{ million}$$

Option 1:

$$\frac{(2000)}{114} = \frac{\cancel{114}(1.018)^t}{\cancel{114}}$$

$$\frac{1000}{57} = (1.018)^t$$

$$\ln\left(\frac{1000}{57}\right) = \ln\left(1.018^t\right)$$

$$\ln\left(\frac{1000}{57}\right) = t \ln(1.018)$$

$$\frac{\ln\left(\frac{1000}{57}\right)}{\ln(1.018)} = t$$

$$\frac{\ln(1000) - \ln(57)}{\ln(1.018)} = t$$

middle of year

$$160.5\ldots = t$$

In what year?

$$\boxed{2160}$$

Option 2:

$$\frac{2000\,\cancel{000}\,\cancel{000}}{114\,\cancel{000}\,\cancel{000}} = \frac{\cancel{114\,000\,000}(1.018)^t}{\cancel{114\,000\,000}}$$

$$\frac{1000}{57} = (1.018)^t$$

$$\vdots$$

$$t \approx 160.5$$

$\rightarrow$ Year: 2160