Recall

$3^x = 27 \qquad x = 3$

$\cancel{3}^x = (\cancel{3})^3$ ← base is the same, don't need them.

$x = 3$

Logarithm Functions

If $x, b \in \mathbb{R}$ and $b \neq 1$, then

$y = \log_b(x)$ is called the logarithmic function with base b.

Logarithm	exponential
$y = \log_b(x)$	$b^y = x$

are equivalent statements

Note: in $y = \log_b(x)$

y = logarithm

b = base

x = argument

Converting from logarithmic to exponential

① $\log_2(32) = 5 \longleftrightarrow 2^5 = 32$

② $\log_{10}\left(\frac{1}{1000}\right) = -3 \longleftrightarrow 10^{-3} = \frac{1}{1000}$

③ $\log_8(1) = 0 \longleftrightarrow 8^0 = 1$

Try

① $\log_3(9) = 2 \longleftrightarrow 3^2 = 9$

② $\log_{10}\left(\frac{1}{100}\right) = -2 \longleftrightarrow 10^{-2} = \frac{1}{100}$

③ $\log_5(1) = 0 \longleftrightarrow 5^0 = 1$

Evaluating Logarithmic Expressions

① $\log_8(64) = x$

$8^x = 64$

$8^x = (8^2)$

$x = 2$

$\log_8(64) = 2$

* Trying to find the exponent

* $a^{-n} = \frac{1}{a^n}$

② $\log_5\left(\frac{1}{125}\right) = -3$

$5^x = \frac{1}{125}$

$5^x = \frac{1}{5^3}$

$5^x = 5^{-3}$

$x = -3$

③ $\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$

$\left(\frac{1}{2}\right)^x = \frac{1}{8}$ $\longrightarrow$ $(2^{-1})^x = \frac{1}{2^3}$

$\left(\frac{1}{2}\right)^x = \frac{1^{3}}{2^3}$

*Quotient Rule for exponents

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3$

$x = 3$

*power of power

$2^{-x} = 2^{-3}$

$-x = -3$

$\frac{-x}{-1} = \frac{-3}{-1}$

$x = 3$

④ $\log_{16}(4) = \frac{1}{2}$

$16^x = 4$ $\longrightarrow$ $16^x = \sqrt{16}$

$(4^2)^x = 4^{1}$

$16^x = 16^{\frac{1}{2}}$

$2x = 1$

$x = \frac{1}{2}$

$x = \frac{1}{2}$

$\log_b(b) = 1$

$x = \log_b(b)$

$b^x = b$

$b^x = b^1$

$x = 1$

* $\log_b(b) = 1$

→ if base of logarithm = argument, then the $\log_b(b) = 1$

$\log_b(b^a) = a$

$x = \log_b(b^a)$

$b^x = b^a$

$x = a$

* $\log_b(b^a) = a$

→ If the argument is a power of the base, the logarithm is the power of the argument.

*Power Rule for Logarithms

Practice

$\log_7(7) = 1$

$\log_2(16) = \log_2(2^4)$

$= 4$

definition:

common logarithm - logarithmic function base 10

- $y = \log(x) \leftrightarrow y = \log_{10}(x)$

Practice

$\log(10{,}000)$

$= \log_{10}(10^4)$ ← 4 zeroes

$= 4$

$\log(0.01)$

$= \log_{10}\left(\frac{1}{100}\right)$

$= \log_{10}(10^{-2})$

$= -2$

$\log\left(\sqrt[4]{10}\right)$

$= \log_{10}\left(10^{\frac{1}{4}}\right)$

$= \frac{1}{4}$

$\log\left(\sqrt[3]{10^2}\right) = \log\left(\left(\sqrt[3]{10}\right)^2\right)$

$= \log_{10}\left(10^{\frac{2}{3}}\right)$

$= \frac{2}{3}$

Properties of Logarithms

Recall $y = \log_b(x) \longleftrightarrow b^y = x$, $x > 0$
$b > 0$
$b \neq 1$

Exponent Rules $\longleftrightarrow$ Logarithm Rules

1. $b^0 = 1$ — $\log_b(1) = 0$
2. $b^1 = b$ — $\log_b(b) = 1$
3. $b^x = b^x$ — $\log_b(b^x) = x$
4. $b^{\log_b(x)} = x$ — $\log_b(b^x) = x$

logarithmic functions and exponential functions are inverses

Practice

a.) $\log_8(8) + \log_8(1)$
$= 1 + 0 = 1$

b.) $\log_{\frac{1}{2}}\left(\left(\frac{1}{2}\right)^x\right) = x$

c.) $10^{\log(x+2)}$

$= 10^{\log_{10}(x+2)}$

$= x+2$

d.) $15^{\log_{15}(7)} = 7$

Recall: $\sin^2(x) = (\sin(x))^2$

$* \log_a^2(x) = (\log_a(x))^2$

$\log(x)^2 = \log(x^2)$

5) $b^M \cdot b^N = b^{M+N}$

Let $b^M = x$

$b^N = y$

So $xy = b^M \cdot b^N$

$xy = b^{M+N}$

$\rightarrow \log_b(xy) = M+N$

$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

"Product Property"

e.g.: $\log(21) = \log(7 \cdot 3)$

$= \log(7) + \log(3)$

$\log(21) = \log(21 \cdot 1)$

$= \log(21) + \log(1)$

$= \log(21) + 0$

$= \log(21)$

* $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

* Quotient Rule for Logarithms

e.g. $\log(3) = \log\left(\frac{21}{7}\right)$

$= \log(21) - \log(7)$

* $\log_b(x^n) = n \log_b(x)$

* Power property of logarithms

e.g. $\log(3^2) = 2\log(3)$

Write logarithmic expression in expanded form.

$$\log_3\left(\frac{xy^3}{z^2}\right)$$

$$= \log_3(xy^3) - \log_3(z^2)$$ Quotient

$$= \left(\log_3(x) + \log_3(y^3)\right) - \log_3(z^2)$$ Product

$$= \log_3(x) + 3\log_3(y) - 2\log_3(z)$$ Power

$$\log\left(\frac{x^4}{yz^3}\right)$$

Quotient

$$\log(x^4) - \log(yz^3)$$

Product

$$\log(x^4) - \left(\log(y) + \log(z^3)\right)$$

$$4\log(x) - \left(\log(y) + 3\log(z)\right)$$ Power

$$4\log(x) - \log(y) - 3\log(z)$$

distributed

$$\log\left(\left(\frac{x^5}{y^4z^5}\right)^8\right)$$

$$8\log\left(\frac{x^5}{y^4z^5}\right)$$ power

$$8\left(\log(x^5)-\log(y^4z^5)\right)$$ quotient

$$8\log(x^5)-8\log(y^4z^5)$$ distributive

$$8\log(x^5)-8\left(\log(y^4)+\log(z^5)\right)$$ quotient

$$8\log(x^5)-8\log(y^4)-8\log(z^5)$$ distributive

$$8\cdot 5\log(x)-8\cdot 4\log(y)-8\cdot 5\log(z)$$

$$40\log(x)-32\log(y)-40\log(z)$$

$$\log\left(\left(\frac{x^5}{y^4 z^5}\right)^8\right) = \log\left(\frac{x^{40}}{y^{32} z^{40}}\right) \rightarrow$$

$$\ldots \rightarrow 40\log(x) - 32\log(y) - 40\log(z)$$

$$\sqrt{y} = y^{\frac{1}{2}}$$

$$\log\left(\frac{x^2\sqrt{y}}{z}\right) =$$

$$+\log(x^2) + \log(\sqrt{y}) - \log(z)$$

if it's in numerator $+$, if in denominator $-$

$$2\log(x) + \log(y^{\frac{1}{2}}) - \log(z)$$

$$2\log(x) + \frac{1}{2}\log(y) - \log(z)$$

Combine to single logarithm

$\sqrt{y} = y^{\frac{1}{2}}$

$9\log(x) + 7\log(z) - \frac{1}{2}\log(y)$

$\log(x^9) + \log(z^7) - \log(\sqrt{y})$

$\log\left(\frac{x^9 z^7}{\sqrt{y}}\right)$

possible route

$+\log\left(y^{-\frac{1}{2}}\right)$

$+\log\left(\frac{1}{\sqrt{y}}\right)$

$b^{\frac{1}{3}} = \sqrt[3]{b}$

$-\frac{7}{2}\log(x) + \frac{3}{2}\log(y) - \frac{5}{2}\log(z) + \frac{9}{2}\log(w)$

$\frac{1}{2}\left(-7\log(x) + 3\log(y) - 5\log(z) + 9\log(w)\right)$

$\frac{1}{2}\log\left(\frac{w^9 y^3}{x^7 z^5}\right)$

$= \log\left(\sqrt{\frac{w^9 y^3}{x^7 z^5}}\right)$

$$\log_3\left(\frac{81}{\sqrt[4]{27}}\right)$$

Quotient

$$=\log_3(81) - \log_3\left(\sqrt[4]{27}\right)$$

$$= \log_3(9^2) - \frac{1}{4}\log_3(27)$$

$$= 2\log_3(9) - \frac{1}{4}\log_3(27)$$

$$= 2\log_3(3^2) - \frac{1}{4}\log_3(3^3)$$

$$= 2\cdot 2\log_3(3) - \frac{1}{4}\cdot 3\log_3(3)$$

$$\log_b(b) = 1$$

$$= 4\left(\frac{4}{4}\right) - \frac{3}{4} =$$

$$= \frac{16}{4} - \frac{3}{4} = \frac{13}{4}$$

$$\log_3\left(\frac{81}{\sqrt[4]{27}}\right)$$

$$= \log_3\left(\frac{3^4}{\sqrt[4]{3^3}}\right)$$

$$= \log_3\left(\frac{3^4}{3^{\frac{3}{4}}}\right)$$

$$= \log_3\left(3^{4-\frac{3}{4}}\right)$$

$$= \log_3\left(3^{\frac{13}{4}}\right)$$

$$= \frac{13}{4}$$

$$\log_b\left(b^x\right) = x$$