Recall the unit circle $x^2+y^2=r^2$

$\sin\theta = y$ $\qquad$ $\csc\theta = \frac{1}{y},\ y\neq 0$

$\cos\theta = x$ $\qquad$ $\sec\theta = \frac{1}{x},\ x\neq 0$

$\tan\theta = \frac{y}{x},\ x\neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\ y\neq 0$

The following can be inferred...

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

These are our "basic trig identities".

$\rightarrow x^2+y^2=1 \leftarrow$ definition of unit circle.

$\rightarrow (\cos\theta)^2+(\sin\theta)^2=1$

$\cos^2\theta + \sin^2\theta = 1$

"Pythagorean Identity"

$\rightarrow 1-\sin^2\theta = \cos^2\theta$

$\rightarrow 1-\cos^2\theta = \sin^2\theta$

Note:

$\cos^2\theta = (\cos\theta)^2$

$\sin^2\theta = (\sin\theta)^2$

Similarly,

$\sin^6\theta = (\sin\theta)^6$

But

$\sin^{-1}\theta \neq (\sin\theta)^{-1}$

Recall $\sin^{-1}\theta = \arcsin\theta$

$\sin^2\theta+\cos^2\theta=1$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$\tan^2\theta + 1 = \sec^2\theta$

$\sin^2\theta+\cos^2\theta=1$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}\right)^2$$

$1 + \cot^2\theta = \csc^2\theta$

Identity: if given two functions $f(x)$ and $g(x)$

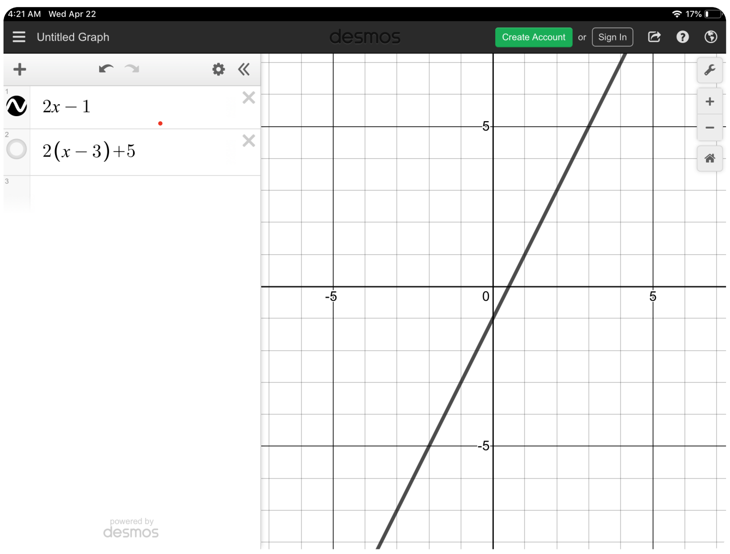
$f(x) = g(x)$ for all values of

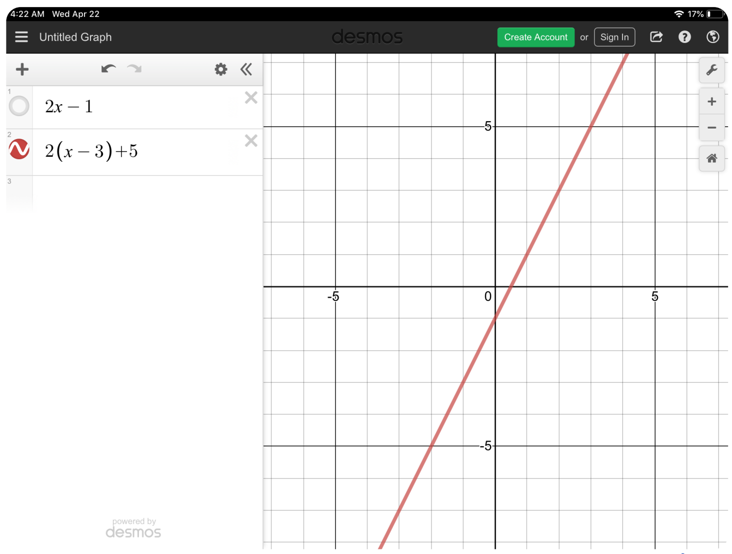
x in both functions domains.

Consider the system of 2 Linear equations

$y = 2x - 1$

$y = 2(x-3) + 5$





Both certainly look equal. Prove it.

$$2x - 1 = 2(x-3) + 5$$

$$2x - 1 = 2x - 6 + 5$$

$$2x - 1 = 2x - 1 \qquad \text{QED.}$$

Notice, we DO NOT MOVE ANYTHING OVER THE EQUAL SIGN.

Our objective for this topic is to prove that these identities are true. We will be using various algebraic manipulations and identity substitutions in order to do so.
I repeat, DO NOT MOVE ANYTHING OVER THE EQUAL SIGN.

Verify that these equations are identities.

* Consider yourselves "plastic surgeons". Your job is to make "ugly" into "pretty".

$$\sin\theta \cot\theta = \cos\theta$$

$$\sin\theta\left(\frac{\cos\theta}{\sin\theta}\right) = \cos\theta$$

$$\cos\theta = \cos\theta \quad \square$$

$$\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$$

$$\cos\theta\sec\theta - \cos^2\theta = \sin^2\theta$$

$$\cos\theta\left(\frac{1}{\cos\theta}\right) - \cos^2\theta = \sin^2\theta$$

$$1 - \cos^2\theta = \sin^2\theta$$

$$(\sin^2\theta + \cos^2\theta) - \cos\theta = \sin^2\theta$$

Not really necessary step, but it helps

$$\sin^2\theta = \sin^2\theta \quad \square$$

$$\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta+\cos^2\theta} = \tan\theta$$

$$\frac{\sin\theta\,(\cos\theta+1)}{\cos\theta\,(\cos\theta+1)} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \tan\theta \qquad \square$$

$$(1+\sin\theta)(1-\sin\theta) = \cos^2\theta$$

$$1-\sin^2\theta = \cos^2\theta$$

$$\cos^2\theta = \cos^2\theta \qquad \text{QED}$$

$$\frac{(\sin\theta+\cos\theta)^2}{\cos\theta} = \sec\theta + 2\sin\theta$$

$$\frac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\cos\theta} = \sec\theta + 2\sin\theta$$

$$\frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\cos\theta} = \sec\theta + 2\sin\theta$$

$$\frac{1 + 2\sin\theta\cos\theta}{\cos\theta} = \sec\theta + 2\sin\theta$$

$$\frac{1}{\cos\theta} + \frac{2\sin\theta\cos\theta}{\cos\theta} = \sec\theta + 2\sin\theta$$

$$\sec\theta + 2\sin\theta = \sec\theta + 2\sin\theta$$

Q.E.D.

$$\frac{\cos^2\theta}{\sin\theta} + \frac{\sin\theta}{1} = \csc\theta$$

LCD

$$\frac{\cos^2\theta}{\sin\theta} + \frac{\sin\theta}{1}\left(\frac{\sin\theta}{\sin\theta}\right) = \csc\theta$$

$$\frac{(\cos^2\theta + \sin^2\theta)}{\sin\theta} = \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$\csc\theta = \csc\theta \qquad \square$$

$$\sin^2\theta \tan^2\theta = \tan^2\theta - \sin^2\theta$$

$$\sin^2\theta \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$\sin^2\theta \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$\sin^2\theta \tan^2\theta = \sin^2\theta \frac{(1-\cos^2\theta)}{\cos^2\theta}$$

$$\sin^2\theta \tan^2\theta = \sin^2\theta \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$\sin^2\theta \tan^2\theta = \sin^2\theta \tan^2\theta \quad \square$$

$$\frac{\cos(t)}{1+\sec(t)} = \frac{1-\cos(t)}{\tan^2(t)}$$

Multiply by the conjugate

$$\frac{\cos(t)}{1+\sec(t)}\left(\frac{1-\sec(t)}{1-\sec(t)}\right) = \frac{1-\cos(t)}{\tan^2(t)}$$

$$\frac{\cos(t)-\cos(t)\sec(t)}{1-\sec^2(t)} = \frac{1-\cos(t)}{\tan^2(t)}$$

$$\frac{\cos(t)-\cos(t)\left(\frac{1}{\cos(t)}\right)}{-(\sec^2(t)-1)} = \frac{1-\cos(t)}{\tan^2(t)}$$

$$-\frac{\cos(t)-1}{\tan^2(t)} = \frac{1-\cos(t)}{\tan^2(t)}$$

$$\frac{1-\cos(t)}{\tan^2(t)} = \frac{1-\cos(t)}{\tan^2(t)} \quad \text{QED}$$

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\sec(x)} = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)}$$

$$\frac{\sec(x)}{\sin(x)}\left(\frac{\sec(x)}{\sec(x)}\right) - \frac{\sin(x)}{\sec(x)}\left(\frac{\sin(x)}{\sin(x)}\right) = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)}$$

$$\frac{\sec^2(x) - \sin^2(x)}{\sin(x)\sec(x)} = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)}$$

$$\frac{(1+\tan^2(x)) - (1-\cos^2(x))}{\sin(x)\left(\frac{1}{\cos(x)}\right)} = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)}$$

$$\frac{1+\tan^2(x) - 1 + \cos^2(x)}{\tan(x)} = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)}$$

$$\frac{\tan^2(x) + \cos^2(x)}{\tan(x)} = \frac{\tan^2(x)+\cos^2(x)}{\tan(x)} \quad \square$$