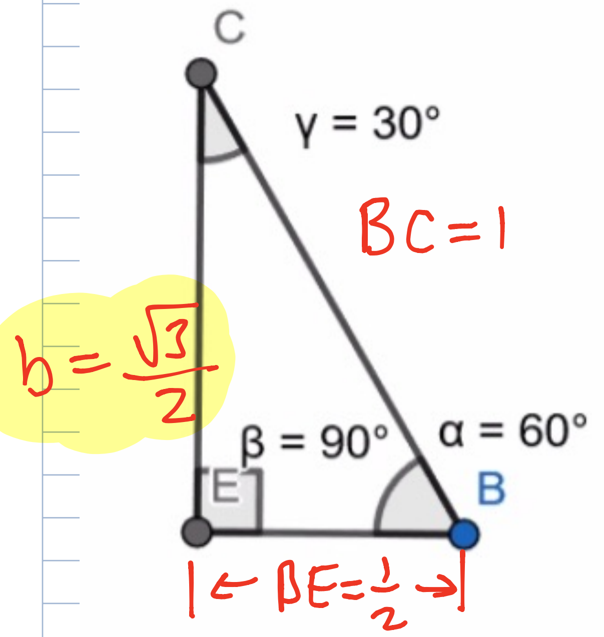


By Pythagorean Theorim

$$a^2 + b^2 = c^2$$

legs hypotenuse

$$\left(\tfrac{1}{2}\right)^2 + b^2 = (1)^2$$

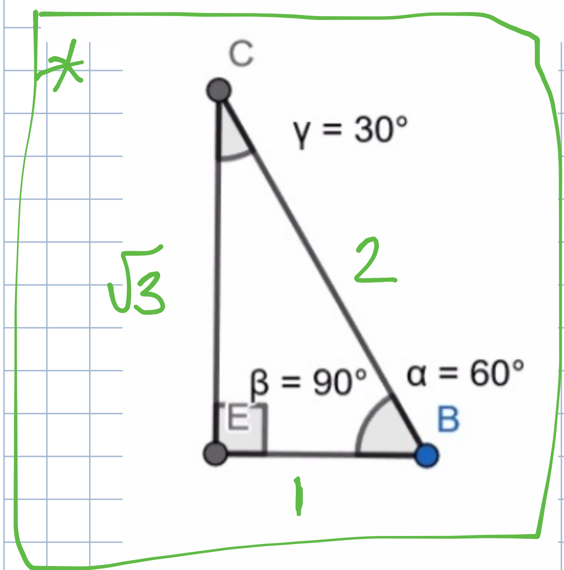
$$\tfrac{1}{4} + b^2 = \cancel{1}\left(\tfrac{4}{4}\right)$$

$$-\tfrac{1}{4} \qquad\qquad -\tfrac{1}{4}$$

$$b^2 = \frac{3}{4}$$

$$b = \pm\frac{\sqrt{3}}{\sqrt{4}}$$

$$b = \pm\frac{\sqrt{3}}{2}$$

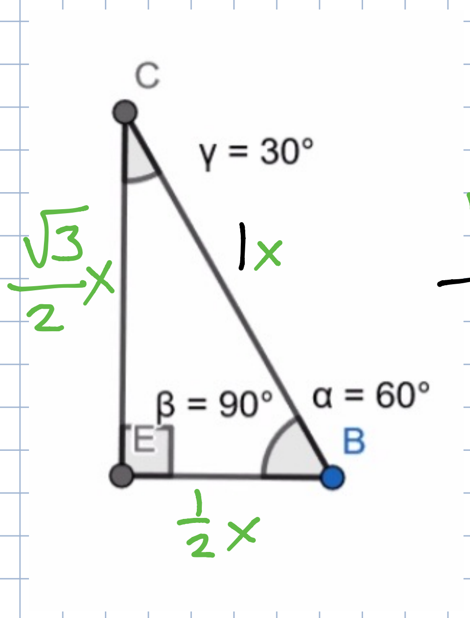


Reject $b = -\frac{\sqrt{3}}{2}$ because length

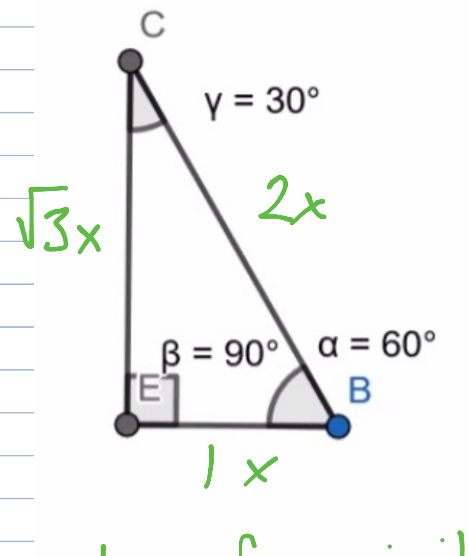
$$b = \frac{\sqrt{3}}{2}$$

Note: similar triangles are proportional

Note: similar triangles are proportional

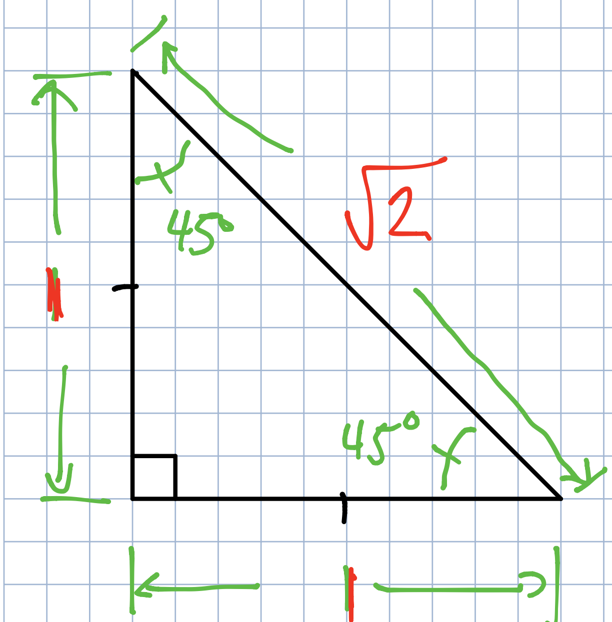


multiply $\longrightarrow$ all sides by 2.



Ratios of 30-60-90△

The side ratios for similar triangles will always remain the same.



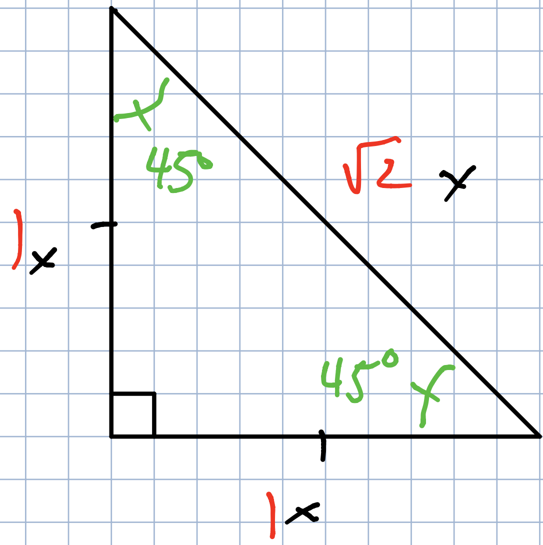
right △

isosceles △ – two sides are congruent

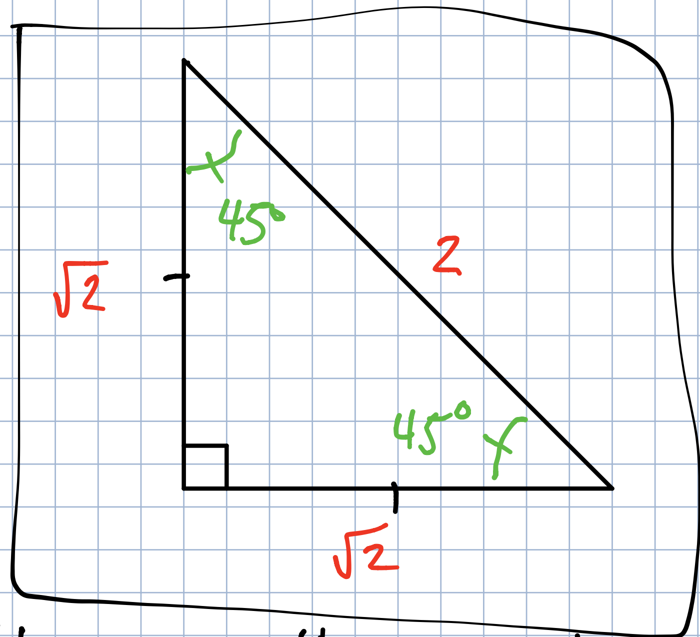
By Pythagorean Theorem

$$a^2+b^2=c^2$$
$$(1)^2+(1)^2=c^2$$
$$1+1=c^2$$
$$2=c^2$$
$$\pm\sqrt{2}=c$$

Reject $c=-\sqrt{2}$

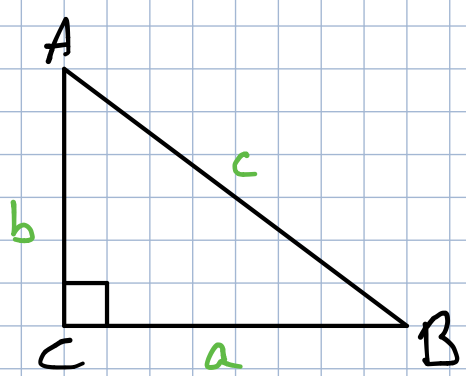


For 45-45-90 △



these are the side ratios

In general, consider a right △



definitions

hypotenuse – the longest side of right △
– opposite right ∡
– in this case side c, $\overline{AB}$

opposite – side not touching the angle in question (NOT THE HYPOTENUSE)

– e.g. side b is opposite ∠B
$\overline{AC}$ is opposite ∠B

side a is opposite ∠A

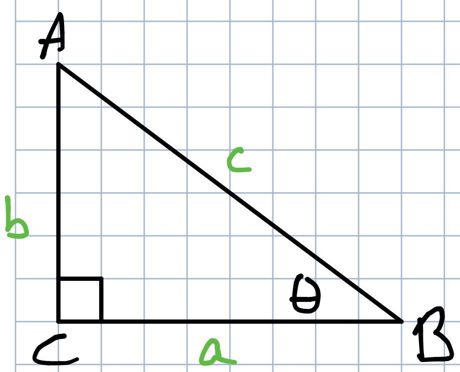
$\overline{BC}$ is opposite $\angle A$

adjacent - side that is touching the angle in question

NOT THE HYPOTENUSE

- e.g. $\overline{BC}$ is adjacent to $\angle B$
  side a is adjacent to $\angle B$

$\overline{AC}$ is adjacent to $\angle A$
side b is adjacent to $\angle A$



define: Assume $\angle\theta$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c} = \frac{AC}{AB}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c} = \frac{BC}{AB}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a} = \frac{AC}{BC}$$

$\overline{AC}$ = line $\overline{AC}$
AC = length of AC

"SOH CAH TOA"

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

$\sin(\angle A) = \frac{a}{c}$

$\cos(\angle A) = \frac{b}{c}$

$\tan(\angle A) = \frac{a}{b}$

$\csc(\angle A) = \frac{c}{a}$

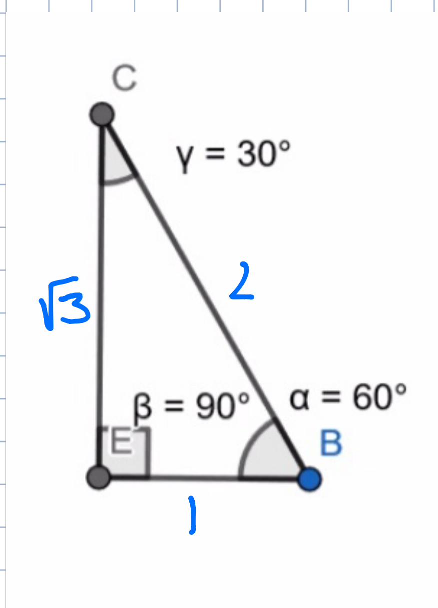
$\sec(\angle A) = \frac{c}{b}$

$\cot(\angle A) = \frac{b}{a}$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\qquad$ $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\qquad$ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\qquad$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$



$\sin(60^\circ) = \dfrac{\sqrt{3}}{2}$ $\qquad$ $\csc(60^\circ) = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\cos(60^\circ) = \dfrac{1}{2}$ $\qquad$ $\sec(60^\circ) = \dfrac{2}{1} = 2$

$\tan(60^\circ) = \dfrac{\sqrt{3}}{1} = \sqrt{3}$ $\qquad$ $\cot(60^\circ) = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

$\sin(30^\circ) = \dfrac{1}{2}$ $\qquad$ $\csc(30^\circ) = \dfrac{2}{1} = 2$

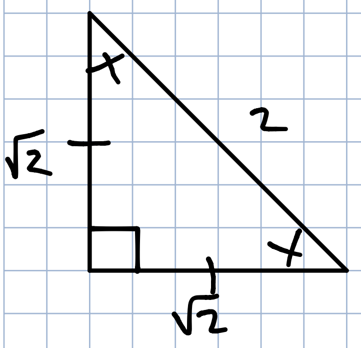
$\cos(30^\circ) = \dfrac{\sqrt{3}}{2}$ $\qquad$ $\sec(30^\circ) = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\tan(30^\circ) = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ $\qquad$ $\cot(30^\circ) = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$\sin(60^\circ) = \cos(30^\circ)$

$\sin(30^\circ) = \cos(60^\circ)$

$$\boxed{\begin{aligned} \cos\theta &= \sin(90-\theta) \\ &= \sin\left(\frac{\pi}{2}-\theta\right) \end{aligned}}$$



$\sin(45^\circ) = \frac{\sqrt{2}}{2}$

$\cos(45^\circ) = \frac{\sqrt{2}}{2}$

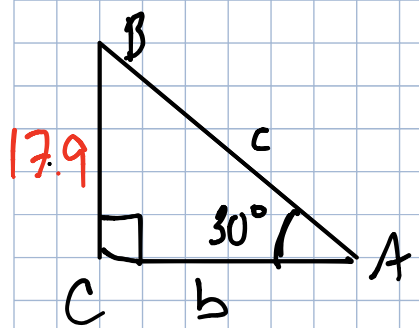
$\tan(45^\circ) = \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\csc(45^\circ) = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\sec(45^\circ) = \sqrt{2}$

$\cot(45^\circ) = 1$

Solving Right Δs



Solve for c

side c is hypotenuse

17.9 is opposite of $\angle A$

$m(\angle A) = 30^\circ$

$\rightarrow \sin(\angle A) = \frac{opp}{hyp}$

$$\frac{\sin(30^\circ)}{1} = \frac{17.9}{c}$$

$$c \cdot \sin(30^\circ) = 17.9$$

$$\frac{c\,\cancel{\sin(30^\circ)}}{\cancel{\sin(30^\circ)}} = \frac{17.9}{\sin(30^\circ)}$$

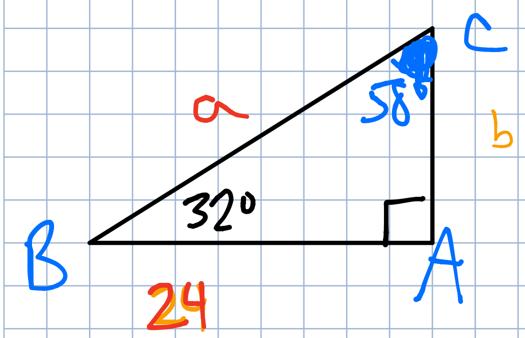
$$c = \frac{17.9}{\sin(30^\circ)}$$

* Note: in this case, we know $\sin(30^\circ) = \frac{1}{2}$

* Only do this if angle is a special angle 0, 30, 45, 60, 90,

$$c = \frac{17.9}{\left(\frac{1}{2}\right)}$$

$$c = 17.9\left(\frac{2}{1}\right) = 35.8$$



Find b

Given: 32°

b is opposite

24 is adjacent

Given side & angle $\rightarrow \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$$\frac{\tan(32°)}{1} = \frac{b}{24}$$

$$24 \cdot \tan(32) = b$$

* Note: We don't know tan(32), use a calculator

* Calculator is set to DEGREES NOT RADIANS

$$24 \cdot \tan(32) = b$$

$$b \approx 38.41$$

Find $m(\measuredangle C)$

$$180° - (90° + 32°) = m(\measuredangle C)$$

sum of angles of Δ

$$180° - 122° = m(\measuredangle C)$$

$$58° = m(\measuredangle C)$$

$$90° - 32° = 58°$$

Find a

Given 32°

$a =$ hypotenuse

$24 =$ adjacent

$$\cos(32^\circ) = \frac{adj}{hyp}$$

$$\cos(32) = \frac{24}{a}$$

$$a\cos(32) = \frac{24}{\cancel{a}} \cdot \cancel{a}$$

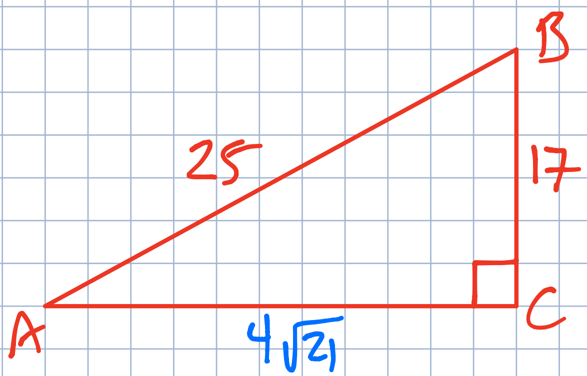
$$\frac{a\cos(32)}{\cos(32)} = \frac{24}{\cos(32)}$$

$$a = \frac{24}{\cos(32)}$$

$$a \approx 28.3$$

rounded

Solve given 2 sides



Solve for AC

$$a^2 + b^2 = c^2$$

$$a^2 + (17)^2 = (25)^2$$

$$a^2 + 289 = 625$$

$$\underline{\quad -289 \qquad -289}$$

$$a^2 = 336$$

$$a = \pm\sqrt{2^4}\sqrt{3 \cdot 7}$$

$$a = \pm\sqrt{(2^2)^2}\sqrt{21}$$

$$a = \pm 2^2\sqrt{21}$$

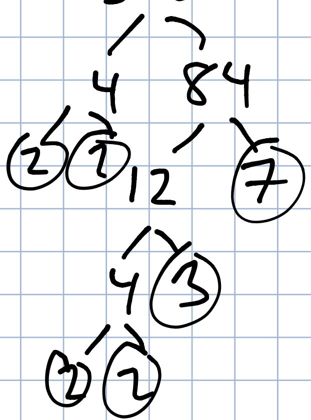
$$a = \pm 4\sqrt{21}$$

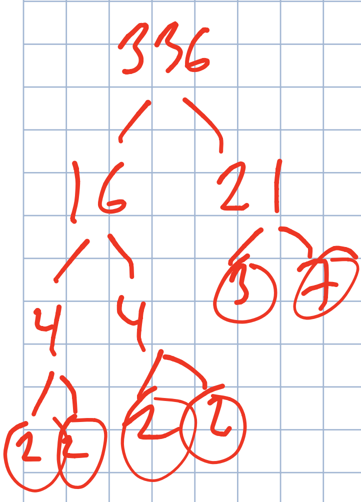
Reject $a = -4\sqrt{21}$

$$a = 4\sqrt{21}$$

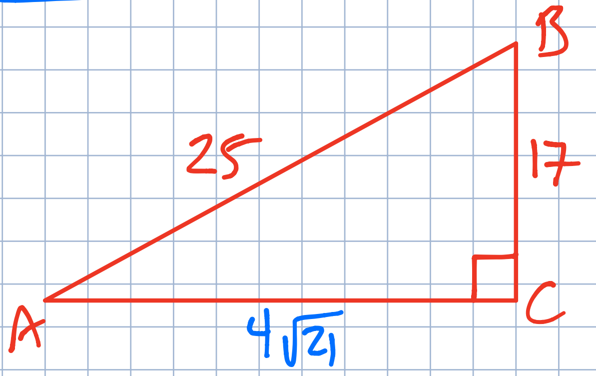
$$AC = 4\sqrt{21}$$

$336 = 2^4 \cdot 3 \cdot 7$





Solve $m(\measuredangle A)$



$\sin(\measuredangle A) = \frac{\text{opp}}{\text{hyp}}$

$\sin(\measuredangle A) = \frac{17}{25}$

$\arcsin(\sin(\measuredangle A)) = \arcsin\left(\frac{17}{25}\right)$

$m(\measuredangle A) = \arcsin\left(\frac{17}{25}\right)$

$m(\measuredangle A) \approx 42.84°$

inverse trig functions

arc- (trig)

undoes a ratio $\longrightarrow$ returns angle

$\arcsin\theta = \sin^{-1}\theta$ = inverse sine function

$\uparrow$

button on calculator

usually $\boxed{\text{2nd}}$ $\boxed{\sin\theta}$

$\tan(\angle A) = \frac{\text{opp}}{\text{adj}}$

$\boxed{\tan^{-1}(17/(4\sqrt{21}))}$

$\tan(\angle A) = \frac{17}{4\sqrt{21}}$

$\arctan(\tan(\measuredangle A)) = \arctan\left(\frac{17}{4\sqrt{21}}\right)$

$m(\measuredangle A) = \arctan\left(\frac{17}{4\sqrt{21}}\right) \approx 42.84°$

$$\cos(\measuredangle A) = \frac{\text{adj}}{\text{hyp}}$$

$$\cos(\measuredangle A) = \frac{4\sqrt{21}}{25}$$

$$\arccos\left(\cos(\measuredangle A)\right) = \arccos\left(\frac{4\sqrt{21}}{25}\right)$$

$$m(\measuredangle A) = \arccos\left(\frac{4\sqrt{21}}{25}\right)$$

$$m(\measuredangle A) = \cos^{-1}\left(\frac{4\sqrt{21}}{25}\right) \approx 42.84°$$

$$m(\angle B) = \arccos\left(\frac{17}{25}\right) \longleftrightarrow \quad \cos(\measuredangle B) = \frac{17}{25}$$

$$= \arcsin\left(\frac{4\sqrt{21}}{25}\right) \longleftrightarrow \quad \sin(\measuredangle B) = \frac{4\sqrt{21}}{25}$$

$$= \arctan\left(\frac{4\sqrt{21}}{17}\right) \longleftrightarrow \quad \tan(\measuredangle B) = \frac{4\sqrt{21}}{17}$$

$$\approx 47.16°$$

$$90 - m(\measuredangle A)$$

$$90 - \arcsin\left(\frac{17}{25}\right)$$