Let us consider 2^x

Difference between x^2 — a number squared

2^x — 2 multiplied by itself x times.

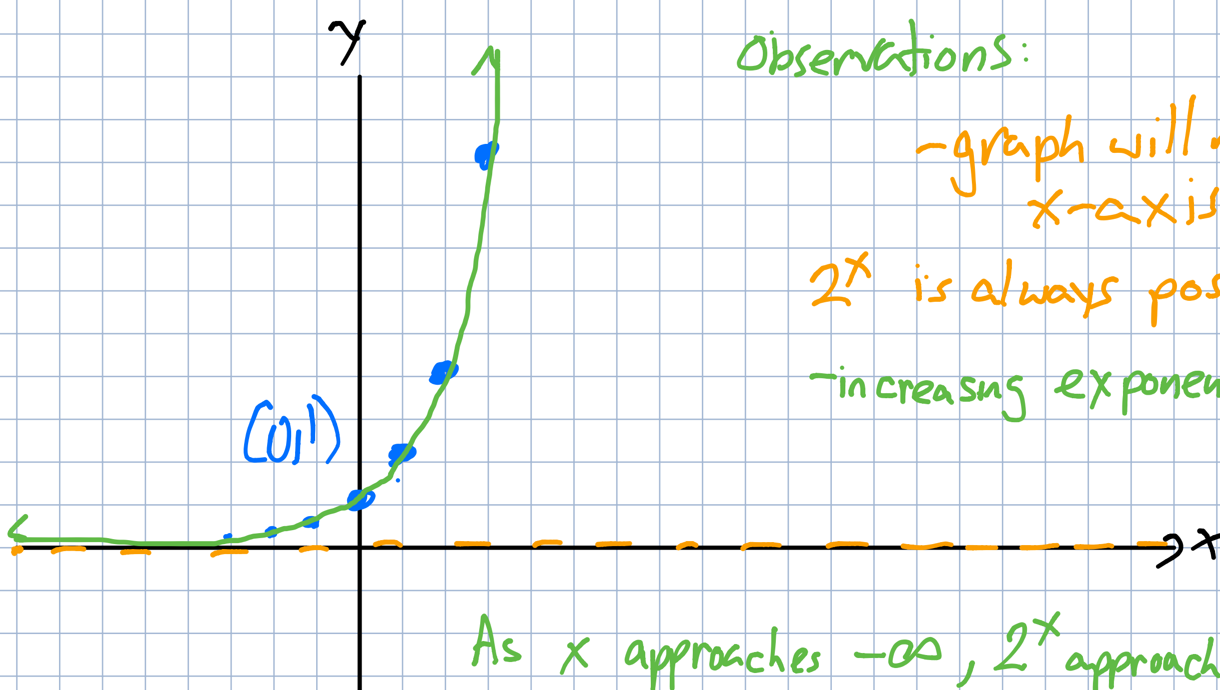
x	-3	-2	-1	0	1	2	3	4	5
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32

$*2$ $*2$ $*2$ $*2$ $*2$ $*2$ $*2$ $*2$

Recall $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Let us plot these points and consider the graph.



Observations:

- graph will never touch x-axis

2^x is always positive

- increasing exponentially

As x approaches $-\infty$, 2^x approaches 0, but $\neq 0$.

x approaches ∞, 2^x approaches ∞

$y = 2^x$ is an example of an exponential function

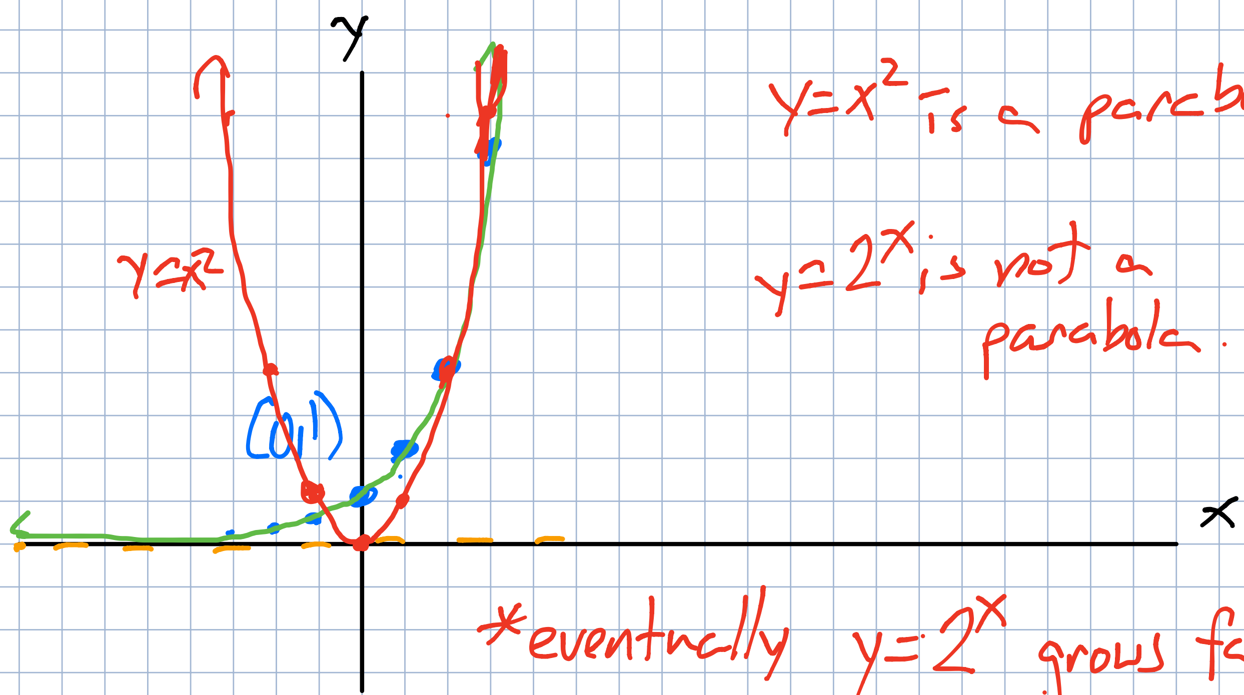
Let $b \in \mathbb{R}^+ \setminus \{1\}$

positive real number, except 1

Then for any $x \in \mathbb{R}$ (real number x),

$y = b^x$ is an exponential function.

Compare the graphs of $y = 2^x$ and $y = x^2$



$y = x^2$ is a parabola

$y = 2^x$ is not a parabola.

*eventually $y = 2^x$ grows faster than $y = x^2$

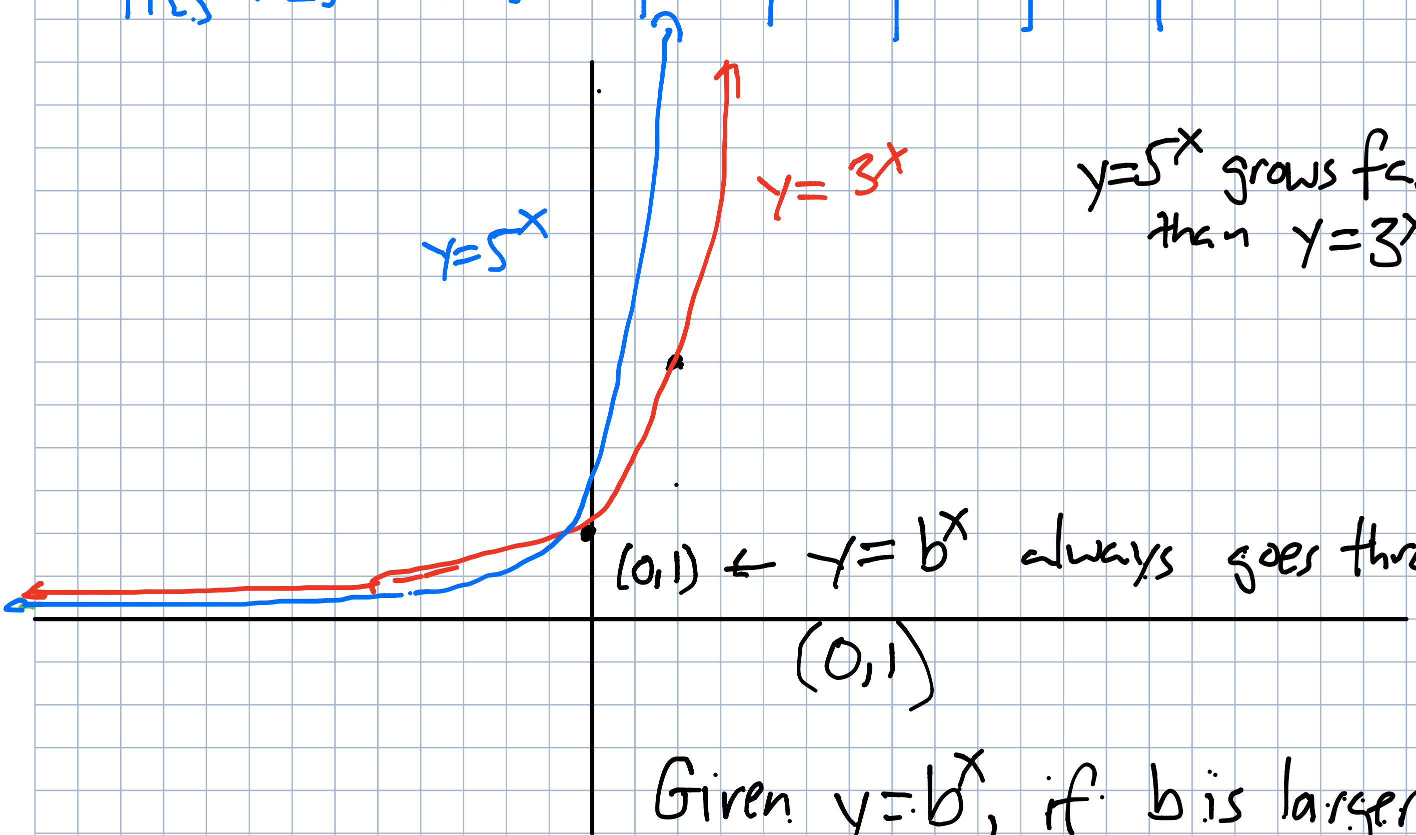
Compare the functions $y=2^x$, $y=3^x$, $y=5^x$

$y=3^x$

x	-3	-2	-1	0	1	2	3	4	5
3^x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

$y=5^x$

x	-3	-2	-1	0	1	2	3	4	5
5^x	$\frac{1}{125}$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125	625	



$y=5^x$ grows faster than $y=3^x$

$(0,1) \leftarrow y=b^x$ always goes through $(0,1)$

Given $y=b^x$, if b is larger, it grows faster.

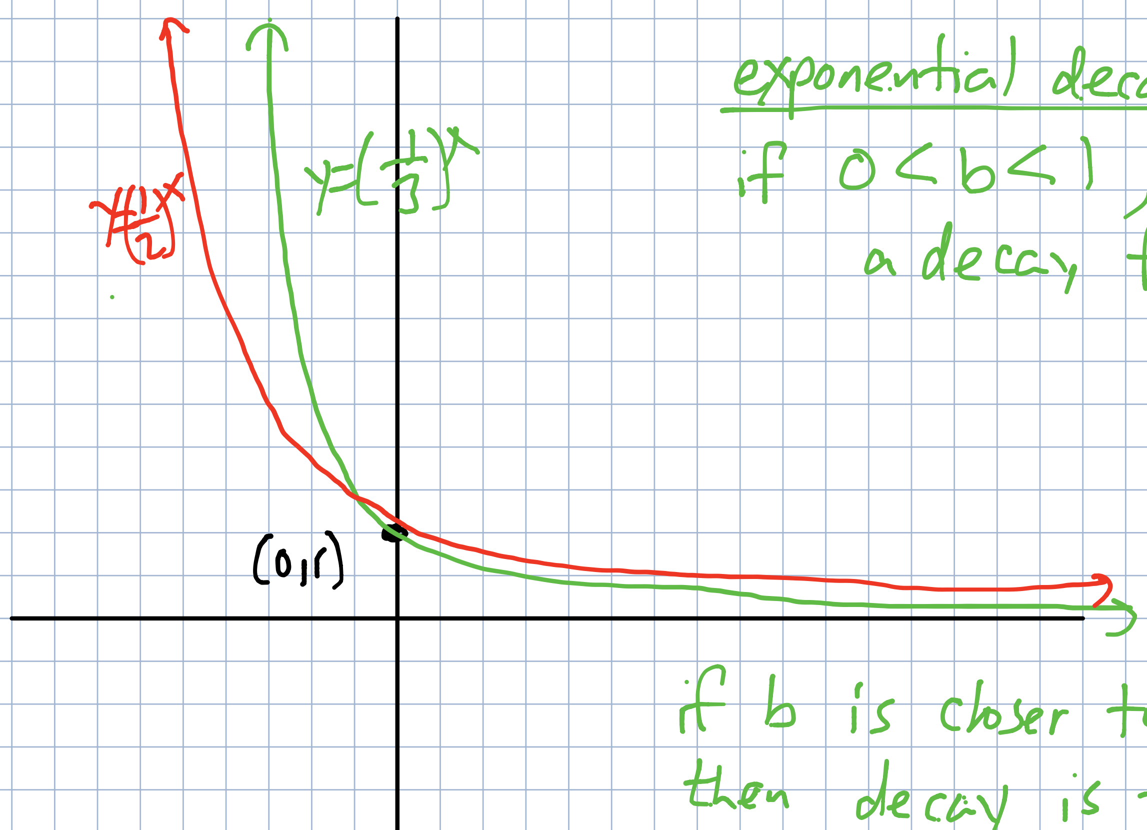
Consider $y=\left(\frac{1}{2}\right)^x$

$y=\left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3	4	5
$\left(\frac{1}{2}\right)^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

$y=\left(\frac{1}{3}\right)^x$

x	-3	-2	-1	0	1	2	3	4	5
$\left(\frac{1}{3}\right)^x$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$



exponential decay

if $0<b<1$, $y=b^x$ is a decay function

if b is closer to 0, then decay is faster.

Combined Observations $y=b^x$

1. $(0,1)$ is the y-int, since $b^0=1$
2. b^x is always greater than 0.
3. $y=b^x$ will never intersect with $y=0$
→ $y=0$ is a horizontal asymptote
4. if $b>1$, exponential growth
if $0<b<1$, exponential decay

General form for exponential growth / decay.

$$y=ab^x$$

$(0,a)$ is y-int

$y=$ final value
$a=$ initial value
$b=$ growth / decay factor
$x=$ time

You will also see these two equations.

$$A=P(1+r)^t$$

$$A=P(1-r)^t$$

if $+$, growth. if $-$, decay

$A=$ final value
$P=$ initial value
$r=$ rate (in decimal)
$t=$ time

Example 4 Applying an Exponential Growth Function

The population of the Bahamas in 2008 was estimated at 321,000 with an annual rate of increase of 1.39%.

a. Find a mathematical model that relates the population of the Bahamas as a function of the number of years since 2008.

b. If the annual rate of increase remains the same, use this model to predict the population of the Bahamas in the year 2016. Round to the nearest thousand.

$P = 321{,}000$

$r = 1.39\%$

$\Rightarrow 0.0139$

$A = ?$

$t =$ years since 2008.

a.) $A = P(1 \pm r)^t$

increase, use $+$

$A = P(1+r)^t$

$A = 321{,}000\,(1+(0.0139))^t$

b.) $t = 2016 - 2008 = 8$ years

$A = 321{,}000\,(1+0.0139)^8$

$A = 321{,}000\,(1.0139)^8$

$\approx 321{,}000\,(1.116762917)$

$\approx 358{,}480.9$

$\approx 358{,}481$ people

$\approx 358{,}000$ people

Skill Practice The population of Colorado in 2000 was approximately 3,700,000 with an annual increase of 2%.

8. Find a mathematical model that relates the population of Colorado as a function of the number of years since 2000.
9. Use this model to predict the population of Colorado in 2016. Round to the nearest thousand.

$A = ?$

$P = 3{,}700{,}000$

$r = 2\% \rightarrow 0.02$

$t =$ years since 2000

$A = P(1+r)^t$

growth

a.) $A = 3{,}700{,}000(1+.02)^t$

$= 3{,}700{,}000(1.02)^t$

b.) $t = 2016 - 2000 = 16$

$A = 3{,}700{,}000(1.02)^{16}$

$A \approx 5{,}079{,}307$ people

$A \approx 5{,}079{,}000$ people