$$2\sin(x) - \sqrt{2} = 0 \quad , \quad x \in [0, 2\pi)$$

$$+\sqrt{2} \quad +\sqrt{2}$$

$$\frac{2\sin(x)}{2} = \frac{\sqrt{2}}{2}$$

$$\sin(x) = \frac{\sqrt{2}}{2}$$

Note: $\sin(x) = + \frac{\sqrt{2}}{2}$

① $\sin(x)$ is positive in QI: $(0, \frac{\pi}{2})$ $(0, 90)$

QII: $(\frac{\pi}{2}, \pi)$ $(90, 180)$

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② Find the reference angle. X_r

$$\sin(X_r) = \left| + \frac{\sqrt{2}}{2} \right|$$

$$\sin(X_r) = \frac{\sqrt{2}}{2}$$

$$\arcsin(\sin(X_r)) = \arcsin\left(\frac{\sqrt{2}}{2}\right)$$

$$X_r = \frac{\pi}{4}$$

---

③ Use reference angle to find solutions in QI and QII.

QI. X_r is assumed to be in QI $\rightarrow$ $x = \frac{\pi}{4}$ is solution

QII. $X_r = \pi - x$

$\frac{\pi}{4} - \pi = -x \rightarrow x = \frac{3\pi}{4}$ is solution.

$$2\cos(x)+\sqrt{3}=0, \quad x\in[0,2\pi)$$

$$-\sqrt{3} \quad -\sqrt{3}$$

$$\frac{2\cos(x)}{2}=\frac{-\sqrt{3}}{2}$$

$$\cos(x)=-\frac{\sqrt{3}}{2}$$

① $\cos(x)<0$ in QII $(\frac{\pi}{2},\pi)$
QIII $(\pi,\frac{3\pi}{2})$

② Find x_r

$$\cos(x_r)=\left|-\frac{\sqrt{3}}{2}\right|$$

$$\cos(x_r)=\frac{\sqrt{3}}{2}$$

$$x_r=\arccos\left(\frac{\sqrt{3}}{2}\right)$$

$$x_r=\frac{\pi}{6}$$

③ Find solutions in QII, QIII

QII. $x_r=\pi-x$

$$\frac{\pi}{6}=\pi-x$$

$$-\pi \quad -\pi$$

$$-\frac{5\pi}{6}=-x$$

$$\boxed{\frac{5\pi}{6}=x}$$

QIII $x_r=x-\pi$

$$\frac{\pi}{6}=x-\pi$$

$$+\pi \quad +\pi$$

$$\boxed{\frac{7\pi}{6}=x}$$

$$\boxed{x\in\left\{\frac{5\pi}{6},\frac{7\pi}{6}\right\}}$$

$$\tan^2\theta - 1 = 0$$

Move-1 across?
Sure...
I feel like doing something else.

$$(\tan\theta)^2 - 1 = 0$$

$$(\tan\theta + 1)(\tan\theta - 1) = 0$$

$\tan\theta + 1 = 0$	$\tan\theta - 1 = 0$
$\tan\theta = -1$	$\tan\theta = 1$
$\tan\theta < 0$ in $Q\,II, Q\,IV$	$\tan\theta > 0$ in $Q\,I, Q\,III$

We need to find 2 reference angles.

$\tan(x_{r_1}) = |-1|$ $\qquad$ $\tan(x_{r_2}) = |1|$

$$\tan(x_{r_1}) = \tan(x_{r_2}) = 1$$

* We are very lucky here. $\rightarrow \quad x_r = \frac{\pi}{4}$

Solutions in $Q\,I, Q\,II, Q\,III, Q\,IV$.

1. $x_r = x = \frac{\pi}{4}$

2. $x_r = \pi - x$

$\rightarrow x = \pi - \frac{\pi}{4}$

$x = \frac{3\pi}{4}$

3. $x_r = x - \pi$

$\rightarrow x = \pi + \frac{\pi}{4}$

$x = \frac{5\pi}{4}$

4. $x_r = 2\pi - x$

$\rightarrow x = 2\pi - \frac{\pi}{4}$

$x = \frac{7\pi}{4}$

$$x \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

$$2\sin(x)\cos(x)+\sqrt{3}\cos(x)=0 \qquad x\in[0,2\pi)$$

Maybe our lives would be easier if we let a = sin(x) and b = cos (x)?

$$2ab+\sqrt{3}\,b=0$$

$$b\left(2a+\sqrt{3}\right)=0$$

$b=0$

$\cos(x)=0$

Quadrantal angles

* $\cos(x)=0$ @ $x=\frac{\pi}{2}+\pi n,\ n\in\mathbb{Z}$

$x=\arccos(0)$

$x=\frac{\pi}{2},\ \frac{3\pi}{2}$

$2a+\sqrt{3}=0$

$a=-\frac{\sqrt{3}}{2}$

$\sin(x)=-\frac{\sqrt{3}}{2}$ → in QIII, QIV

→ $x_r=\arcsin\left(\frac{\sqrt{3}}{2}\right)$

$x_r=\frac{\pi}{3}$

→ QIII $x_r=x-\pi$

$\frac{\pi}{3}=x-\pi$

$+\pi \qquad +\pi$

$\frac{4\pi}{3}=x$

→ QII. $x_r=\pi-x$

$\frac{\pi}{3}=\pi-x$

$-\pi \qquad -\pi$

$-\frac{2\pi}{3}=-x$

$x=\frac{2\pi}{3}$

$$\boxed{\therefore\ x\in\left\{\frac{\pi}{2},\frac{2\pi}{3},\frac{4\pi}{3},\frac{3\pi}{2}\right\}}$$

$$2\cos^2(x) - 7\sin(x) - 5 = 0 \qquad , x \in [0, 2\pi)$$

HUGE PROBLEM... Looks Like a quadratic but cos(x) and sin(x)?
Perhaps there is an identity that would help us.

$$2(1-\sin^2(x)) - 7\sin(x) - 5 = 0$$

$$2 - 2\sin^2(x) - 7\sin(x) - 5 = 0$$

$$-2\sin^2(x) - 7\sin(x) - 3 = 0$$

Let u= sin(x). Never let x=sin(x). x is already being used.

$$-2u^2 - 7u - 3 = 0$$

$$-(2u^2 + 7u + 3) = 0$$

$$2u^2 + 7u + 3 = 0$$

$$(u+3)(2u+1) = 0$$

$u + 3 = 0$

$u = -3$

🤔 $\sin(x) = -3$?

reject

$-1 \leq \sin(x) \leq 1$

$2u + 1 = 0$

$u = -\frac{1}{2}$

$\sin(x) = -\frac{1}{2} \rightarrow$ QIII, QIV

$\rightarrow \sin(x_r) = \left|-\frac{1}{2}\right|$

$x_r = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\rightarrow$ in QIII

$x_r = x - \pi$

$\rightarrow x = \pi + \frac{\pi}{6}$

$x = \frac{7\pi}{6}$

$\rightarrow$ in QIV

$x_r = 2\pi - x$

$\rightarrow x = 2\pi - \frac{\pi}{6}$

$x = \frac{11\pi}{6}$

$$\boxed{\therefore x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$$

$$3\tan^2(x) - 4\sqrt{3}\tan(x) + 3 = 0$$

$$3u^2 - 4\sqrt{3}u + 3 = 0$$

$$3u^2 - 4\sqrt{3}u = -3$$

$$u^2 - \frac{4\sqrt{3}}{3}u = -1$$

completed the square for practice.

$$u^2 - \frac{4\sqrt{3}}{3}u + \left(\frac{\cancel{4}2\sqrt{3}}{\cancel{6}3}\right)^2 = -1 + \left(\frac{2\sqrt{3}}{3}\right)^2$$

$$\left(u - \left(\frac{2\sqrt{3}}{3}\right)\right)^2 = -1 + \left(\frac{4\cdot\cancel{3}}{3\cdot\cancel{3}}\right)$$

$$\left(u - \frac{2\sqrt{3}}{3}\right)^2 = \frac{1}{3}$$

$$u - \frac{2\sqrt{3}}{3} = \pm\frac{\sqrt{3}}{3}$$

$$u = \frac{2\sqrt{3}}{3} \pm \frac{\sqrt{3}}{3}$$

$$u = \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3}$$

$$u = \sqrt{3}$$

$$\tan(x) = \sqrt{3} \to QI, QIII$$

$$\to \tan(x_{r_1}) = \sqrt{3}$$

$$x_{r_1} = \frac{\pi}{3}$$

$$\to QI, \quad x = \frac{\pi}{3}$$

$$QIII, \quad x = \frac{4\pi}{3}$$

$$u = \frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3}$$

$$u = \frac{\sqrt{3}}{3}$$

$$\tan(x) = \frac{\sqrt{3}}{3} \to QI, QIII$$

$$\to \tan(x_{r_2}) = \frac{\sqrt{3}}{3}$$

$$x_{r_2} = \frac{\pi}{6}$$

$$\to QI, x = \frac{\pi}{6}$$

$$QIII, x = \frac{7\pi}{6}$$

$$\boxed{\therefore x \in \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\right\}}$$