

(Amorj)

$$\tan(21) = \frac{\text{opp}}{\text{adj}}$$

(Leanne)

$$101 + 21 = 122$$

$$180 - 122 = 58^\circ$$

(Whitney)

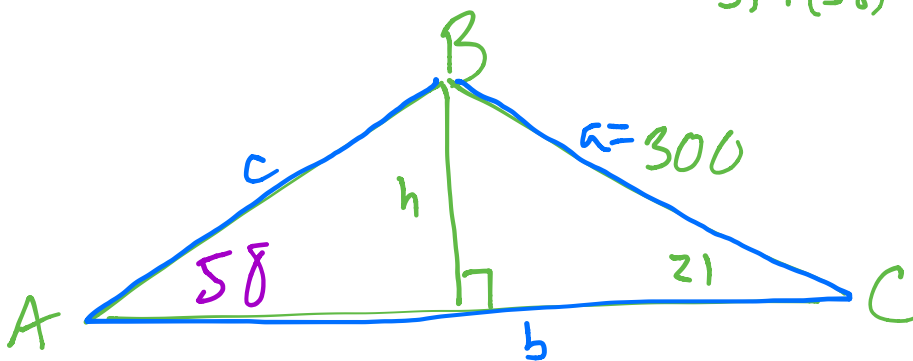
Found 58°

Made 2 right triangles

$$\sin(21) = \frac{\text{height}}{\text{hypotenuse}}$$

Solved for height

$$\sin(58) = \frac{\text{height}}{AB}$$



$$\sin(21) = \frac{h}{300}$$

$$h = 300 \sin(21)$$

$$\sin(58) = \frac{h}{AB}$$

$$\sin(58) = \frac{300 \sin(21)}{AB} \dots$$

$$\frac{300 \sin(21)}{\sin(58)} = AB$$

$$\sin(21) = \frac{h}{300}$$

$$h = 300 \sin(21)$$

$$\sin(58) = \frac{h}{AB}$$

$$\sin(58) = \frac{h}{c}$$

$$h = c \cdot \sin(58)$$

$$300 \sin(21) = c \cdot \sin(58)$$

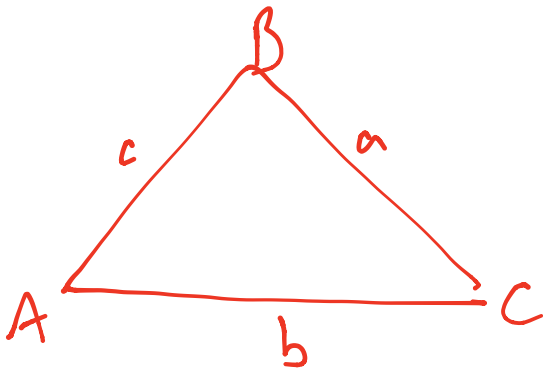
$$a \sin(\angle C) = c \sin(\angle A)$$

Trying to solve for side c.

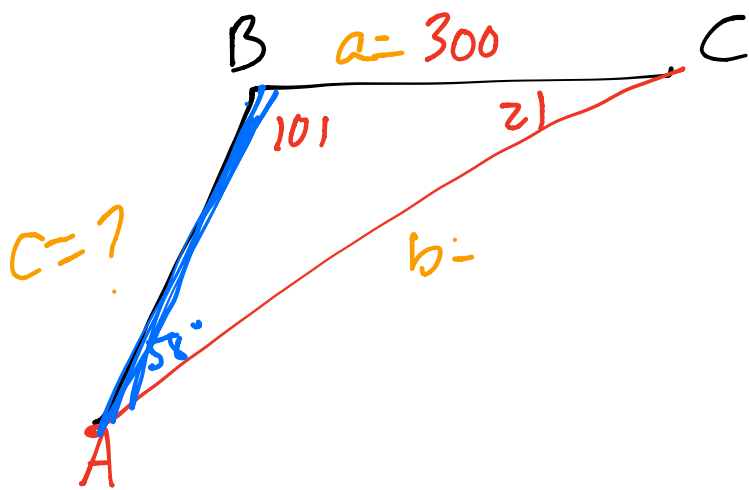
$$\frac{a \sin(\angle C)}{\sin(\angle A)} = c$$

$$\frac{a}{\sin(\angle A)} = \frac{c}{\sin(\angle C)} = \frac{b}{\sin(\angle B)}$$

* The law of sines



$$\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b} = \frac{\sin(\angle C)}{c}$$



AB is opposite

$\angle C$

c is opposite
of $\angle C$

Needed to find
3rd angle

$$\frac{\sin(58)}{300} = \frac{\cancel{\sin(101)}}{\cancel{b}} = \frac{\sin(21)}{c}$$

$$\frac{\sin(58)}{300} = \frac{\sin(21)}{c} \leftarrow$$

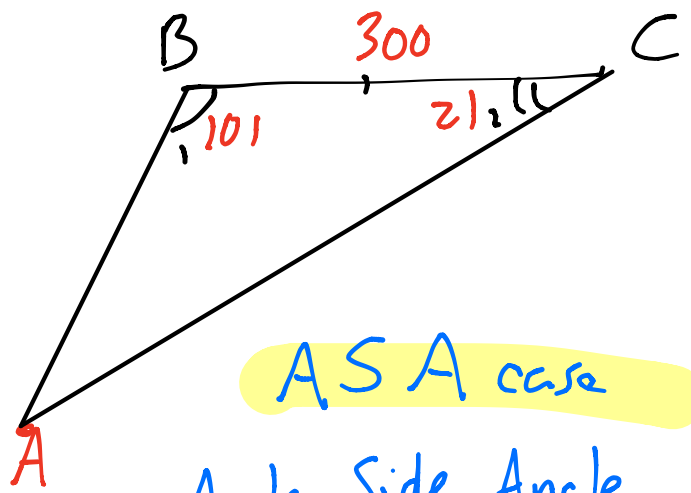
$$\frac{300 \sin(21)}{\sin(58)} = \frac{c \cancel{\sin(58)}}{\cancel{\sin(58)}}$$

$$\frac{300 \sin(21)}{\sin(58)} = c$$

$$126.77 \approx c$$

$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$$

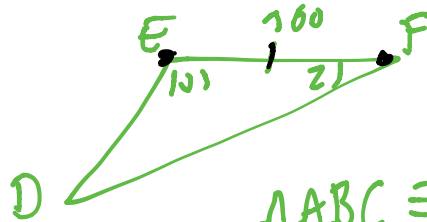
$$\frac{\sin(58)}{300} = \frac{\sin(21)}{c}$$



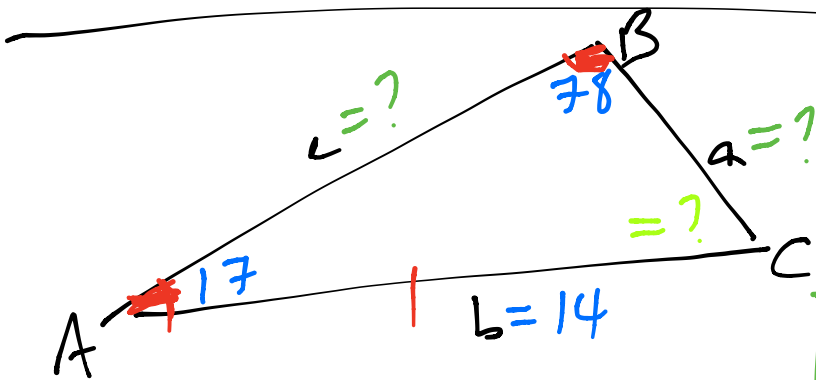
ASA case

Angle Side Angle

Recall



$\triangle ABC \cong \triangle DEF$
by ASA



$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\frac{\sin(17)}{a} = \frac{\sin(78)}{14}$$

$$a \frac{\sin(78)}{\sin(78)} = \frac{14 \sin(17)}{\sin(78)}$$

$$a \approx 4.185$$

AAS case

Angle Angle side

$$m(\angle C) = 180 - (78 + 17) = 85^\circ$$

$$\frac{\sin(78)}{14} = \frac{\sin(85)}{c}$$

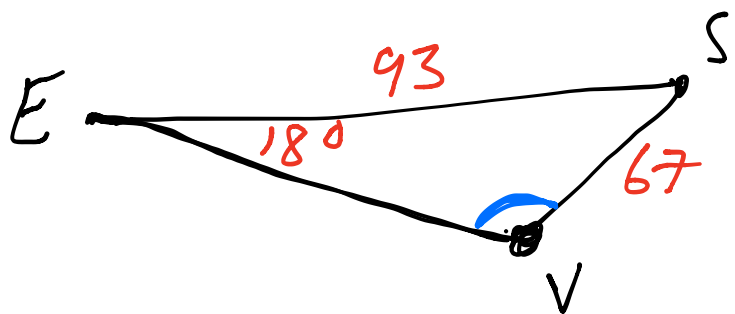
$$c \sin(78) = 14 \sin(85)$$

$$c = \frac{14 \sin(85)}{\sin(78)} \approx 14.258$$

← real value

← approximated value

AAS case
ASA case \searrow LAW of Sines



SSA case
Side Side Angle

Find $\angle V$

$$\frac{\sin(18)}{67} = \frac{\sin(\angle V)}{93}$$

$$67 \sin(\angle V) = 93 \sin(18)$$

$$\sin(\angle V) = \frac{93 \sin(18)}{67}$$

Trig Equation

$$\sin^{-1}(\sin(\angle V)) = \sin^{-1}\left(\frac{93 \sin(18)}{67}\right)$$

$$m(\angle V) \approx 25.4^\circ \leftarrow \text{in QI } (0, 90^\circ)$$

$$\text{in QII } (90^\circ, 180^\circ) \approx 180^\circ - 25.4^\circ \approx 154.6^\circ$$

Try to find $\angle S$, Test both cases

$$\text{Let } \angle V = 25.4 \rightarrow 180 - (25.4 + 18) = 136.6^\circ$$

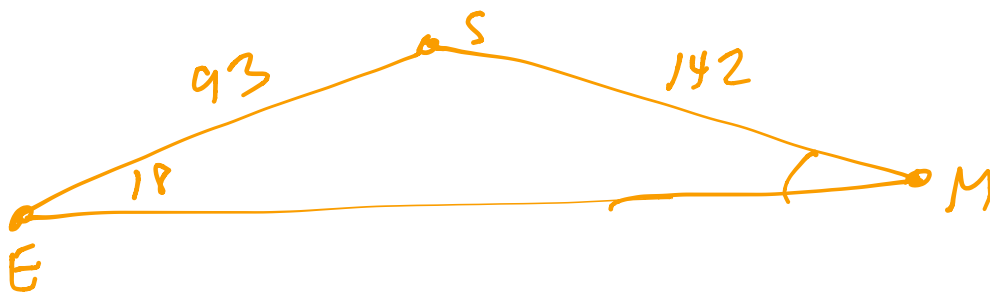
$\rightarrow 25.4$ is a solution

$$\text{Let } \angle V = 154.6 \rightarrow 180 - (154.6 + 18) = 7.4^\circ$$

$\rightarrow 154.6$ is also a solution

$$\therefore m(\angle V) = 25.4^\circ, 154.6^\circ$$

SSA case gives 0 solutions
1 solution
2 solutions



$$\frac{\sin(18)}{142} = \frac{\sin(M)}{93}$$

$$\therefore \sin(M) = \frac{93 \sin(18)}{142} \approx 11.7^\circ \text{ in QI}$$

$$\approx 180 - 11.7 \approx 168.3^\circ \text{ in QII}$$

Try to find $\angle S$

$$\angle M = 11.17^\circ \rightarrow 180 - (18 + 11.17) = 150.3$$

$$\angle M = 168.3^\circ \rightarrow 180 - (18 + 168.3) = -6.3$$

↑
rejected

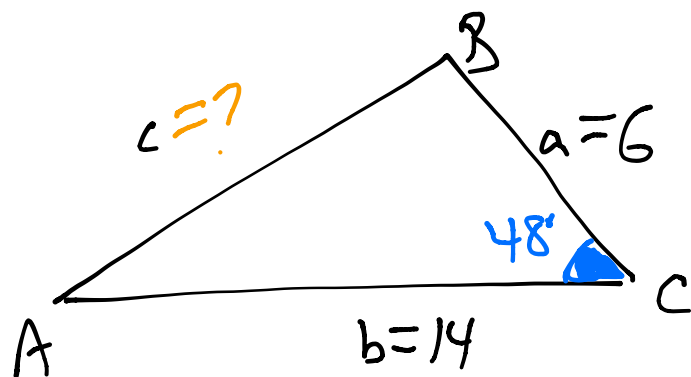
↑
negative angle

$$\therefore \angle M = 11.17^\circ$$

SSA case: 0 solutions

$$\sin \theta > 1 \quad \text{or} \quad \sin \theta < -1$$

No solutions



SAS case
use law of cosines

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos(\angle C)$$

$$c^2 = (6)^2 + (14)^2 - 2(6)(14) \cos(48)$$

$$c^2 = 36 + 196 - 168 \cos(48)$$

$$c^2 = 232 - 168 \cos(48)$$

$$c = \sqrt{232 - 168 \cos(48)} \leftarrow \text{Reject negative side length}$$

$$c \approx 119.586$$

Law of Cosines (for a side)

$$c^2 = a^2 + b^2 - 2ab \cos(\angle C)$$

$$a^2 = b^2 + c^2 - 2bc \cos(\angle A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(\angle B)$$

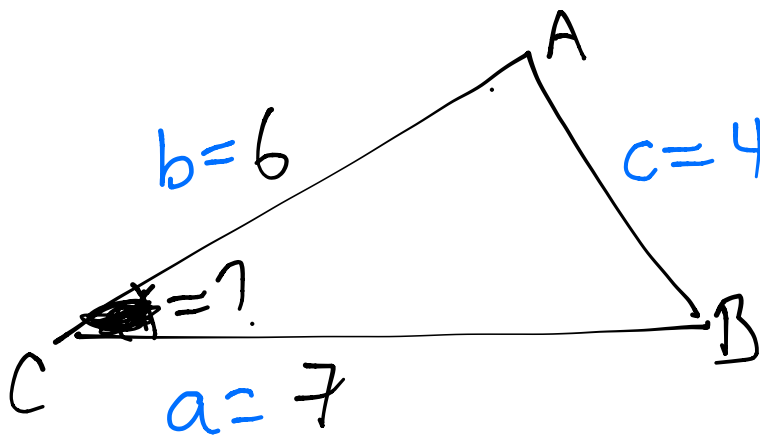
Law of Cosines (for angle

$$c^2 = (a^2 + b^2) - 2ab \cos(\angle C)$$

$$\frac{c^2 - (a^2 + b^2)}{-2ab} = \frac{-2ab \cos(\angle C)}{-2ab}$$

$$\frac{c^2 - (a^2 + b^2)}{-2ab} = \cos(\angle C)$$

$$\downarrow$$
$$\cos(\angle C) = \frac{a^2 + b^2 - c^2}{2ab}$$



SSS case

Use law of cosines

$$\cos(\angle C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(\angle C) = \frac{(7)^2 + (6)^2 - (4)^2}{2(7)(6)}$$

$$\cos(\angle C) = \frac{23}{28}$$

$$m(\angle C) = \cos^{-1}\left(\frac{23}{28}\right)$$

$$m(\angle C) \approx 34.772^\circ \text{ in QI } (0, 90^\circ)$$

don't need to \rightarrow QIV $(270^\circ, 360^\circ)$
above 180°