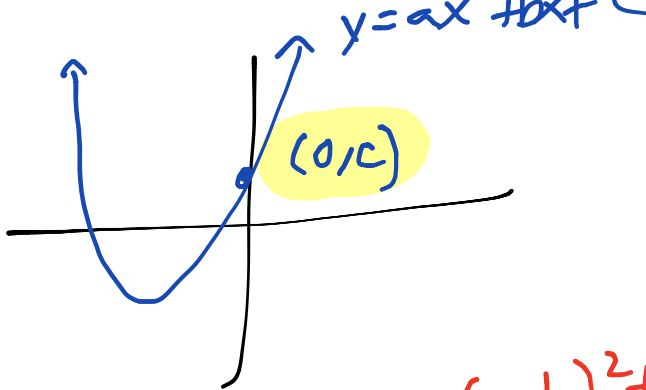
Forms of a quadratic function

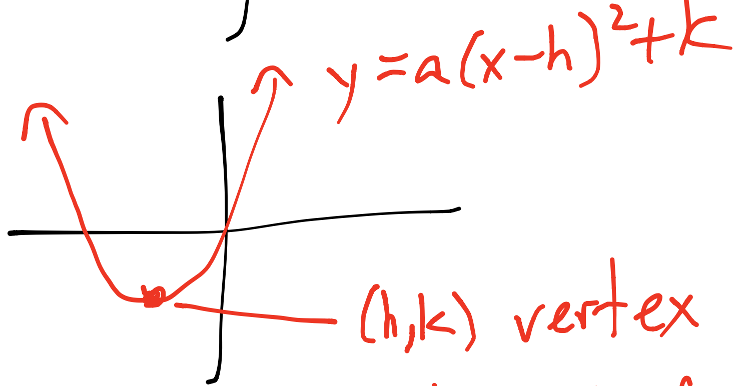
standard form

$y = ax^2 + bx + c$



vertex form

$y = a(x-h)^2 + k$



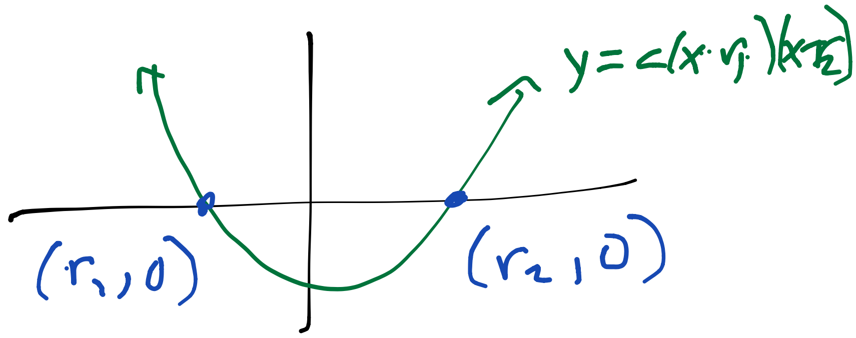
$x=h$ axis of symmetry

$k=$ extremum

maximum or
minimum value

roots form

$y = a(x-r_1)(x-r_2)$



r_1, r_2 are solutions
to $ax^2+bx+c=0$
$a(x-h)^2+k=0$
$a(x-r_1)(x-r_2)=0$

Converting from one form to the other.

$y = ax^2 + bx + c$ to $y = a(x - r_1)(x - r_2)$

standard   roots

e.g

$y = 2x^2 - 12x + 10 \rightarrow y = 2(x^2 - 6x + 5)$

$(0, 10)$

$y = 2(x-5)(x-1)$

$ac = 5 = \underline{-5} \cdot \underline{-1}$

$b = -6 = \underline{-1} + \underline{-5}$

$(5, 0), (1, 0)$

* factor to go from standard to roots

Going from standard form to vertex form.

$y = ax^2 + bx + c$ to $y = a(x-h)^2 + k$

* Recall

$ax^2 + bx + c = 0 \longrightarrow a(x-h)^2 + k = 0$

complete the square

e.g. $y = 2x^2 - 12x + 10$

$\quad -10 \qquad\qquad -10$

$y - 10 = 2x^2 - 12x$

$\frac{y-10}{2} = \frac{\cancel{2}(x^2 - 6x)}{\cancel{2}}$

* When completing the square. keep x^2.

$\frac{y-10}{2} + \left(\frac{6}{2}\right)^2 = x^2 - 6x + \left(\frac{6}{2}\right)^2 \leftarrow$ * $+\left(\frac{b}{2}\right)^2$

$\frac{y-10}{2} + 9 = x^2 - 6x + 9$

$\frac{y-10}{2} + 9 = (x-3)^2$

$\frac{y-10}{2} = (x-3)^2 - 9$

$\frac{y}{2} - \cancel{\frac{10}{2}}\,5 = (x-3)^2 - 9$

$\frac{y}{2} = (x-3)^2 - 4$

$y = 2((x-3)^2 - 4)$

$y = 2(x-3)^2 - 8$

$y = 2(x-3)^2 + (-8)$

vertex: $(3, -8)$

one-sided

$$y = 2x^2 - 12x + 10$$

$$y = (2x^2 - 12x) + 10$$

$$y = 2(x^2 - 6x) + 10$$

$$y = 2\left(x^2 - 6x + \left(\frac{6}{2}\right)^2\right) + 10 - 2\left(\frac{6}{2}\right)^2$$

$$y = 2(x - 3)^2 + 10 - 18$$

$$y = 2(x - 3)^2 - 8$$

vertex $(3, -8)$

vertex form : $y = a(x - h)^2 + k$

(h, k) is vertex

Converting via completing the square

$$y = ax^2 + bx + c$$

$$y - c = ax^2 + bx$$

$$\frac{y - c}{a} = x^2 + \frac{b}{a}x$$

$$\frac{y-c}{a} + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$$

$$\frac{y-c}{a} + \left(\frac{b}{2a}\right)^2 = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{y-c}{a} = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$$

$$y - c = a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + c$$

$$y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(\frac{4ac - b^2}{4a}\right)$$

vertex formula: (axis of symmetry)

finding $x = h = -\dfrac{b}{2a}$ *

Finding k in vertex: — replace x with h in

$$y = ax^2 + bx + c$$

$$\longrightarrow k = ah^2 + bh + c$$

$$\text{—or} \quad k = \frac{4ac - b^2}{4a}$$

$$\text{vertex } (h, k) = \left(-\frac{b}{2a}, f(h)\right) = \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

Parabola Lab

$(h,k) = (1,1) \longrightarrow y = a(x-1)^2 + 1$

$(r_1, 0) = (2,0)$

$(r_2, 0) = (0,0)$

$\longrightarrow y = a(x-2)(x-0)$

* Find a.

From vertex form

$y = a(x-1)^2 + 1$

$\boxed{\text{Try } (1,1) \leftarrow \text{vertex}}$

$(1) = a((1)-1)^2 + 1$

$1 = a(0)^2 + 1$

$\cancel{1} = 0a + \cancel{1}$

$-1 \qquad -1$

$0 = 0$ ← didn't solve for a

* Try $(2,0)$ ← not vertex

$y = a(x-1)^2 + 1$

$(0) = a(2-1)^2 + 1$

$0 = a + 1$

$a = -1$

*Recall $y = mx + b$

slope $= 6$ point given $(1,3)$

need to find b

$y = mx + b$

$(3) = (6)(1) + b$ ...

Replace b in $y = mx + b$

*Replace a in

$y = a(x-h)^2 + k$

or

$y = a(x-r_1)(x-r_2)$

$\rightarrow y = -1(x-1)^2 + 1$

$\rightarrow y = -1(x-2)(x-0)$

From roots form

$y = a(x-2)(x-0)$

Try $(0,0)$ ← x-intercept

$0 = a(0-2)(0-0)$

$0 = a(-2)(0)$

$0 = 0$

didn't solve for a

Pick $(1,1)$ ← not the root

$1 = a(1-2)(1-0)$

$1 = a(-1)(1)$

$1 = -a$

$a = -1$

→ $y = -1(x-2)(x-0)$

→ $y = -1(x-1)^2 + 1$