$$\sqrt[3]{x} = 5$$

$$\left(\sqrt[3]{x}\right)^3 = (5)^3$$

$$x = 125$$

* Radical Equation
– has one or more radicals

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$$\sqrt{x} = -7$$

$$\left(\sqrt[2]{x}\right)^2 = (-7)^2$$

$$x = 49$$

possible solution

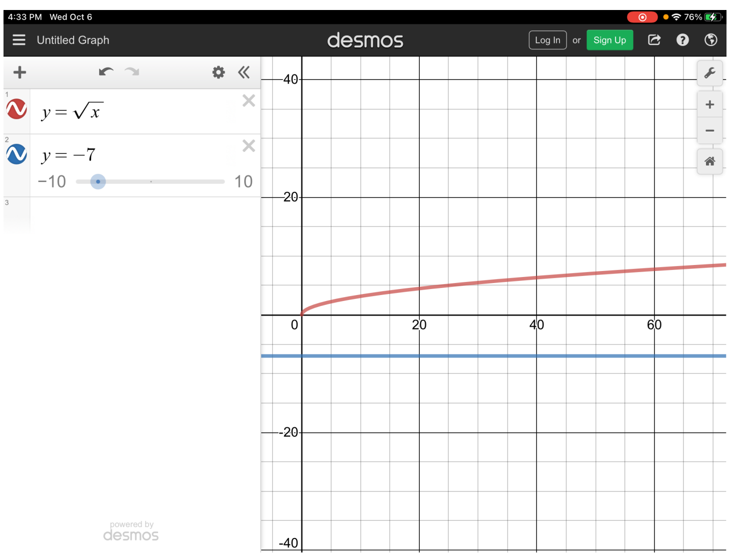
Check $x = 49$

$$\sqrt{49} = -7 \rightarrow x \neq 49$$

$$7 = -7$$

* no sign, assume positive square root

$$\boxed{\text{no solution}}$$




Note:we are multiplying both sides of an equation by an algebraic factor.
A new equation is formed and often, there are new solutions to the equation.
Thus, it is important to check all possible solutions.

$$\sqrt{x} - 3 = 2$$
$$+3 \quad +3$$
$$\sqrt{x} = 5$$
$$(\sqrt{x})^2 = (5)^2$$
$$x = 25$$
possible solution

Check $x=25$

$$\sqrt{25} - 3 = 2$$
$$5 - 3 = 2$$
$$2 = 2$$

$\boxed{\therefore\ x=25 \text{ is the solution}}$

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$$\sqrt{6x+55} + 5 = 0$$
$$-5 \quad -5$$
$$\sqrt{6x+55} = -5$$
$$\left(\sqrt{6x+55}\right)^2 = (-5)^2$$
$$6x+55 = 25$$
$$-55 \quad -55$$
$$6x = -30$$
$$\frac{6x}{6} = -\frac{30}{6}$$
$$x = -5$$

Check $x = -5$

$$\sqrt{6(-5)+55} + 5 = 0$$
$$\sqrt{-30+55} + 5 = 0$$
$$\sqrt{25} + 5 = 0$$
$$5 + 5 = 0$$
$$10 \neq 0 \ ?$$

$\rightarrow x = -5$ is not a solution

$\boxed{\therefore\ \text{no solution}}$

$$y + 2\sqrt{4y-3} = 3$$
$$-y \qquad\qquad -y$$
$$2\sqrt{4y-3} = 3-y$$

$$(2\sqrt{4y-3})^2 = (3-y)^2$$

* power of product

$$2^2(\sqrt{4y-3})^2 = (3-y)(3-y)$$

$$4(4y-3) = 9-6y+y^2$$

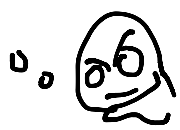
$$16y-12 = y^2-6y+9$$
$$-16y+12 \qquad -16y+12$$

$$0 = y^2-22y+21$$
$$0 = (y-1)(y-21)$$

*a=1

b = -22 = -1 + -21

c = 21 = -1 · -21



$$y-1=0 \qquad\qquad y-21=0$$
$$y=1 \qquad\qquad y=21$$

Need to check $y+2\sqrt{4y-3}=3$

$y=1$

$$(1)+2\sqrt{4(1)-3}=3$$
$$1+2\sqrt{1}=3$$
$$1+2=3$$
$$3=3 \checkmark$$

$y=21$

$$(21)+2\sqrt{4(21)-3}=3$$
$$21+2\sqrt{84-3}=3$$
$$21+2\sqrt{81}=3$$
$$21+2(9)=3$$

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$$2\sqrt{4y-3} = 3-y$$

$$\frac{\cancel{2}\sqrt{4y-3}}{\cancel{2}} = \frac{3-y}{2}$$

$$\sqrt{4y-3} = \frac{3-y}{2}$$

$$(\sqrt{4y-3})^2 = \left(\frac{3-y}{2}\right)^2$$

$$\frac{4y-3}{1} = \frac{9-6y+y^2}{4}$$

$$4(4y-3) = y^2-6y+9$$

⋮

$\rightarrow y = 1$ is a solution

$21 + 18 = 3$

$39 \neq 3$

$\rightarrow y = 21$ is not a solution

$\boxed{\therefore\ y = 1 \text{ is the only solution}}$

$$\sqrt{a^2+8a+16} = a+6$$

$\vdots$

$$\sqrt{(a+4)^2} = a+6$$

* $\cancel{a+4 = a+6}$

$\cancel{-a-4 \quad -a-4}$

$\cancel{0 = 2}$

→ No solution?

$$|a+4| = a+6$$

$a+4=a+6$ positive

$\vdots$

we just did this

$-(a+4)=a+6$ negative

$-a-4 = a+6$

$+a-6 \quad +a-6$

$-10 = 2a$

$$\boxed{-5 = a}$$

still check

$$a^2+8a+16 = (a+6)^2$$

$$a^2+8a+16 = a^2+12a+36$$

$$-a^2-8a-16 \qquad -a^2-8a-16$$

$$0 = 4a+20$$

$$-20 \qquad -20$$

$$-20 = 4a$$

$$\frac{-20}{4} = \frac{4a}{4}$$

$$a = -5 \;?$$

Check $a=-5$

$$\sqrt{(-5)^2+8(-5)+16} = (-5)+6$$

$$\sqrt{25-40+16} = 1$$

$$1 = 1 \checkmark$$

$$\boxed{\therefore a=-5 \text{ is the solution}}$$

$$\sqrt{2p^2-12p+10} = p-1$$

$$2p^2-12p+10 = p^2-2p+1$$

$$-p^2+2p-1 \qquad -p^2+2p-1$$

$$p^2-10p+9 = 0$$

$$(p-1)(p-9) = 0$$

$p-1=0$ or $p-9=0$

$+1 \; +1 \qquad +9 \; +9$

$p=1 \qquad p=9$

Check $\sqrt{2p^2-12p+10} = p-1$

$p=1$ ✓

$p=9$ ✓

$$\sqrt{2(9)^2-12(9)+10} = 9-1$$

$$\sqrt{64} = 8$$

$$8 = 8 \checkmark$$

$\therefore$ $p=1$ and $p=9$ are both solutions

$$\sqrt{2x+6} - \sqrt{x+4} = 1$$
$$+\sqrt{x+4} \quad +\sqrt{x+4}$$
$$\sqrt{2x+6} = 1+\sqrt{x+4}$$
$$\left(\sqrt{2x+6}\right)^2 = \left(1+\sqrt{x+4}\right)^2$$

$$(a+b)^2 = a^2+2ab+b^2$$
$$\left(1+\sqrt{x+4}\right)^2$$

$$2x+6 = 1+2\sqrt{x+4}+x+4$$
$$2x+6 = x+5+2\sqrt{x+4}$$
$$-x-5 \quad -x-5$$
$$x+1 = 2\sqrt{x+4}$$

$$(x+1)^2 = \left(2\sqrt{x+4}\right)^2$$
$$(x+1)(x+1) = 2^2\cdot\left(\sqrt{x+4}\right)^2$$
$$x^2+2x+1 = 4(x+4)$$
$$x^2+2x+1 = 4x+16$$
$$-4x-16 \quad -4x-16$$
$$x^2-2x-15 = 0$$
$$(x-5)(x+3) = 0$$

$x+3=0$ or $x-5=0$

$$-3 \quad -3 \qquad\qquad +5 \quad +5$$
$$x=-3 \qquad\qquad x=5$$

Check $\sqrt{2x+6} - \sqrt{x+4} = 1$

$x = -3$ ✗

reject

$x = 5$ ✓

$\boxed{\therefore\ x = 5 \text{ is the solution}}$