$$\frac{7-5i}{2+3i}\cdot\frac{2-3i}{2-3i}=\frac{(7-5i)(2-3i)}{(2+3i)(2-3i)}$$

$$(a+bi)(a-bi)=a^2+b^2$$

	2	$-3i$
7	14	$-21i$
$-5i$	$-10i$	$15i^2$ -15

$$14-15-10i-21i$$

$$-1-31i$$

$$\frac{7-5i}{2+3i}$$

$$=\frac{-1-31i}{(2)^2+(3)^2}$$

$$=\frac{-1-31i}{4+9}$$

$$=\frac{-1-31i}{13}$$

$$=-\frac{1}{13}-\frac{31}{13}i$$

$$\frac{4-i}{-3i}\cdot\frac{+3i}{+3i}=\frac{(4-i)(3i)}{(-3i)(3i)}$$

$*0-3i$

conjugate

$0+3i$

$$=\frac{12i-3i^2}{-9i^2}$$

$$=\frac{12i+3}{9}$$

$$= \frac{3}{9} + \frac{12}{9}i$$

$$\frac{4-i}{-3i} \qquad = \frac{1}{3} + \frac{4}{3}i$$

$$\frac{4-i}{-3i}\left(\frac{i}{i}\right) = \frac{(4-i)i}{-3i^2}$$

$$= \frac{4i - i^2}{3}$$

$$= \frac{4i - (-1)}{3}$$

$$= \frac{4i + 1}{3}$$

$$= \frac{1}{3} + \frac{4}{3}i$$

$x^2 = 81$

$x^2 - 81 = 0$

$(x+9)(x-9) = 0$

$x+9=0$ or $x-9=0$

$-9 \quad -9$ $\qquad$ $+9 \quad +9$

$\boxed{x=-9}$ $\qquad$ $\boxed{x=9}$

$\boxed{x = 9, -9}$

$x^2 = 81$

$\sqrt{x^2} = \pm\sqrt{81}$

$x = \pm 9$

Using Square Root Property

$x^2 = 25$

$\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

There are two solutions

$4x^2 = 9$

$\frac{4x^2}{4} = \frac{9}{4}$

$x^2 = \frac{9}{4}$

$\sqrt{x^2} = \pm\sqrt{\frac{9}{4}}$

$x = \pm\frac{3}{2}$

$4x^2 = 9$

$\sqrt{4x^2} = \pm\sqrt{9}$

$2x = \pm 3$

$\frac{2x}{2} = \pm\frac{3}{2}$

$x = \pm\frac{3}{2}$

Square Root Property

For any real number k, $k \in \mathbb{R}$

if $x^2 = k$,

then $x = -\sqrt{k}$

or

$x = +\sqrt{k}$

$x = \pm\sqrt{k}$

$$x^2 + 25 = 0$$
$$-25 \quad -25$$
$$x^2 = -25$$
$$\sqrt{x^2} = \pm\sqrt{-25}$$
$$x = \pm\sqrt{25}\sqrt{-1}$$
$$x = \pm 5i$$

* No real solution

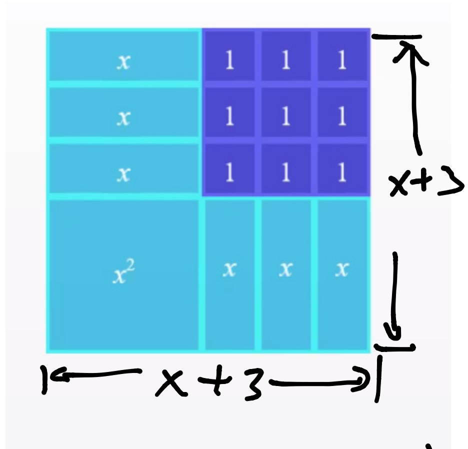
$$(w+3)^2 = 20$$
$$\sqrt{(w+3)^2} = \pm\sqrt{20}$$
$$w+3 = \pm\sqrt{4}\sqrt{5}$$
$$w+3 = \pm 2\sqrt{5}$$
$$-3 \quad -3$$
$$w = -3 \pm 2\sqrt{5}$$

$$(t-5)^2 = -18$$
$$\sqrt{(t-5)^2} = \pm\sqrt{-18}$$
$$t-5 = \pm\sqrt{9}\sqrt{2}\,i$$
$$t-5 = \pm 3\sqrt{2}\,i$$
$$+5 \quad +5$$
$$t = 5 \pm 3\sqrt{2}\,i$$

$5-3\sqrt{2}i$, $5+3\sqrt{2}i$

$$x^2+6x-13=0$$
$$\quad +13 \;+13$$

$$x^2+6x = 13$$
$$x^2+6x+9 = 13+9$$
$$(x+3)^2 = 22$$
$$\sqrt{(x+3)^2} = \pm\sqrt{22}$$
$$x+3 = \pm\sqrt{22}$$
$$x = -3\pm\sqrt{22}$$

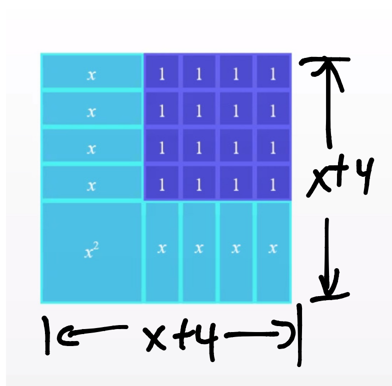


split the 6 in half → 3 and 3

add square $9 = 3^2$

Complete the square

$$x^2+8x+16 = (x+4)^2$$



Re allot the 8x
"split them in 2" → 4 by 4

Add a square $16 = 4^2$

Given x^2+bx. To complete the square
Add $\left(\frac{b}{2}\right)^2$

Verify: $x^2+bx+\left(\frac{b}{2}\right)^2$ is a perfect square

$(a+b)^2=a^2+2ab+b^2$

$$x^2+bx+\left(\frac{b}{2}\right)^2=(x)^2+2\left(\frac{b}{2}\right)(x)+\left(\frac{b}{2}\right)^2$$

$$bx = \cancel{2}\left(\frac{b}{\cancel{2}}\right)x$$

$$bx = bx$$

$\therefore$ $x^2+bx+\left(\frac{b}{2}\right)^2$ is a perfect square

* $x^2+bx+\left(\frac{b}{2}\right)^2 = \left(x+\frac{b}{2}\right)^2$
* $x^2-bx+\left(-\frac{b}{2}\right)^2 = \left(x-\frac{b}{2}\right)^2$

$$x^2+6x = 13$$

$$x^2+6x+\left(\frac{6}{2}\right)^2 = 13+\left(\frac{6}{2}\right)^2$$

$$\left(x+\frac{6}{2}\right)^2 = 13+9$$

$$(x+3)^2 = 22$$

$$x+3 = \pm\sqrt{22}$$

$$x = -3\pm\sqrt{22}$$

$$y^2 - 5y + 25 = 0$$

$$y^2 - 5y = -25$$

$$y^2 - 5y + \left(-\frac{5}{2}\right)^2 = -25 + \left(-\frac{5}{2}\right)^2$$ ← complete the square

$$\left(y - \frac{5}{2}\right)^2 = -25\left(\frac{4}{4}\right) + \frac{25}{4}$$ ← factored the square

$$\left(y - \frac{5}{2}\right)^2 = \frac{-100}{4} + \frac{25}{4}$$

$$\left(y - \frac{5}{2}\right)^2 = -\frac{75}{4}$$

$$y - \frac{5}{2} = \pm\sqrt{-\frac{75}{4}}$$

$$y - \frac{5}{2} = \pm\frac{\sqrt{25}\sqrt{3}}{\sqrt{4}}i$$

$$y - \frac{5}{2} = \pm\frac{5\sqrt{3}}{2}i$$

$$y = \frac{5}{2} \pm \frac{5\sqrt{3}}{2}i$$

$$2b^2 - 12b = 5$$ ← we want a coefficient of 1 for b^2

$$2(b^2 - 6b) = 5$$

$$b^2 - 6b = \frac{5}{2}$$ ← divide both sides by 2

$$b^2 - 6b + \left(-\frac{6}{2}\right)^2 = \frac{5}{2} + \left(-\frac{6}{2}\right)^2$$

$$(b-3)^2 = \frac{5}{2} + 9\left(\frac{2}{2}\right)$$

$$(b-3)^2 = \frac{5}{2} + \frac{18}{2}$$

$$(b-3)^2 = \frac{23}{2}$$

$$b - 3 = \pm\frac{\sqrt{23}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$b - 3 = \pm\frac{\sqrt{46}}{2}$$

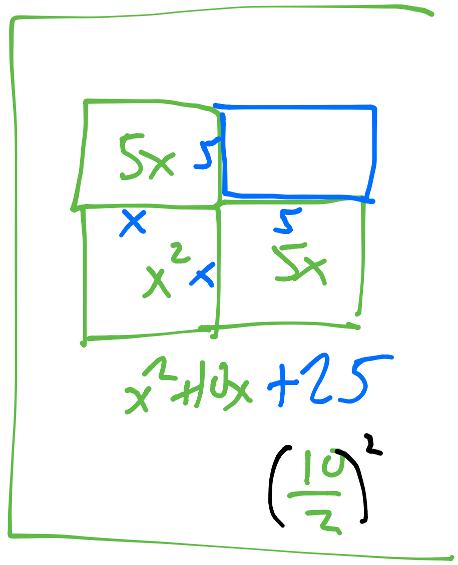
$$b = 3 \pm \frac{\sqrt{46}}{2}$$

$$3x^2 - 30x - 5 = 0$$
$$+5 \qquad +5$$

$$3x^2 - 30x = 5$$

$$3(x^2 - 10x) = 5$$

$$x^2 - 10x = \frac{5}{3}$$



$$x - 10x + \left(-\frac{10}{2}\right)^2 = \frac{5}{3} + 25\left(\frac{3}{3}\right)$$

$$(x-5)^2 = \frac{5}{3} + \frac{75}{3}$$

$$\left(x - \frac{10}{2}\right)^2$$

$$(x-5)^2 = \frac{80}{3}$$

$$\sqrt{(x-5)^2} = \pm\sqrt{\frac{80}{3}}$$

$$\frac{\sqrt{80}}{\sqrt{3}} = \frac{\sqrt{16}\sqrt{5}}{\sqrt{3}} = \frac{4\sqrt{5}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{4\sqrt{15}}{3}$$

$$x - 5 = \pm\frac{4\sqrt{15}}{3}$$
$$+5 \qquad +5$$

$$x = 5 \pm \frac{4\sqrt{15}}{3}$$

Consider quadratic equation: Complete the square.

$a, b, c \in \mathbb{R}$

$a \neq 0$

$$ax^2 + bx + c = 0$$

$$ - c \quad - c$$

$$ax^2 + bx = -c$$

$$\frac{ax^2 + bx}{a} = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$* \quad \frac{\frac{b}{a}}{\frac{2}{1}} = \frac{b}{a} \cdot \frac{1}{2} = \frac{b}{2a}$$

Perfect square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Quadratic Formula

Any solution of $ax^2+bx+c=0$, $a,b,c\in\mathbb{R}$, $a\neq 0$

has the form $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$3x^2-30x-5=0$$

$a=3$

$b=-30$

$c=-5$

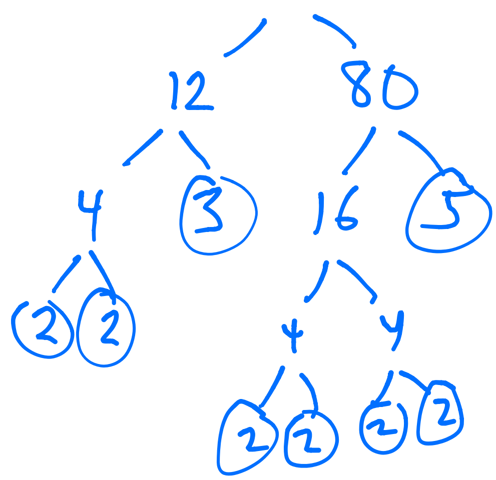
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-30)\pm\sqrt{(-30)^2-4(3)(-5)}}{2(3)}$$

$$=\frac{30\pm\sqrt{900+60}}{6}$$

$$=\frac{30\pm\sqrt{960}}{6}$$

$960=2^6\cdot3\cdot5$



$$=\frac{30\pm\sqrt{2^6}\sqrt{15}}{6}$$

$$=\frac{30\pm8\sqrt{15}}{6}$$

$$=\frac{\cancel{2}(15\pm4\sqrt{15})}{\cancel{2}\cdot3}$$

$$\frac{30}{6}\pm\frac{8\sqrt{15}}{6}$$

$$= \frac{15 \pm 4\sqrt{15}}{3}$$

$$= \frac{15}{3} \pm \frac{4\sqrt{15}}{3}$$

$$= 5 \pm \frac{4\sqrt{15}}{3}$$

$$\boxed{5 \pm \frac{4\sqrt{15}}{3}}$$