imaginary numbers:

$$i = \sqrt{-1}$$

Given $\sqrt{-b}$ for $b > 0$
"b is a positive number":

Then $\sqrt{-b} = \sqrt{b} \cdot \sqrt{-1}$

$$\sqrt{-b} = \sqrt{b}\ i \qquad \text{or } i\sqrt{b}$$

e.g. $\sqrt{-64} = \sqrt{64}\sqrt{-1} = 8i$

$-\sqrt{-4} = -\sqrt{4}\sqrt{-1} = -2i$

$\sqrt{-29} = \sqrt{29}\ i$

$\sqrt{-50} = \sqrt{25}\sqrt{2}\sqrt{-1} = 5\sqrt{2}\ i$

Keep i outside the radical

$$\sqrt{2i} \neq \sqrt{2}\ i$$

$$\sqrt{2}\sqrt{i} \neq \sqrt{2}\ i$$

$$\sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$$

$$\frac{\sqrt{-100}}{\sqrt{-25}} = \frac{10i}{5i} = 2$$

$$\sqrt{-5}\sqrt{-5} = \sqrt{5}\,i \cdot \sqrt{5}\,i = \sqrt{25}\,i^2 = 5i^2 = 5(-1) = -5$$

$$\sqrt{-25}\sqrt{-9} = 5i \cdot 3i = 15i^2 = 15(-1) = -15$$

$i^1 = \sqrt{-1} = i$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^3 \cdot i = 1$

$= (-i)i$

$= -i^2$

$= -(-1)$

$i^5 = i^4 i = i^1$

$= 1i$

$i^6 = i^5 i = i^2 = -1$

$= i\,i$

$i^7 = i^3 = -i$

$i^8 = i^4 = 1$

$i^9 = i^1$

$i^{10} = i^2 = -1$

$i^{11} = i^3 = -i$

$i^{12} = i^4 = 1$

* repeats after every 4th power of i

$i^0 = i^4 = i^8 = i^{12} = \ldots = i^{4n+0} = 1$

$i^1 = i^5 = i^9 = i^{13} = \ldots = i^{4n+1} = i$

$i^2 = i^6 = i^{10} = i^{14} = \ldots = i^{4n+2} = -1$

$i^3 = i^7 = i^{11} = i^{15} = \ldots = i^{4n+3} = -i$

* all even powers of i result in ± 1

* all odd powers of i result in $\pm i$

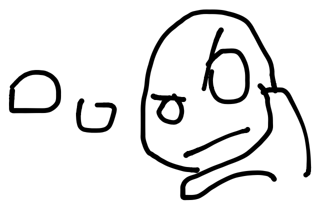
$i^{27} = i^{3} = -i$

$27 \div 4 = 6 \text{ R} 3$

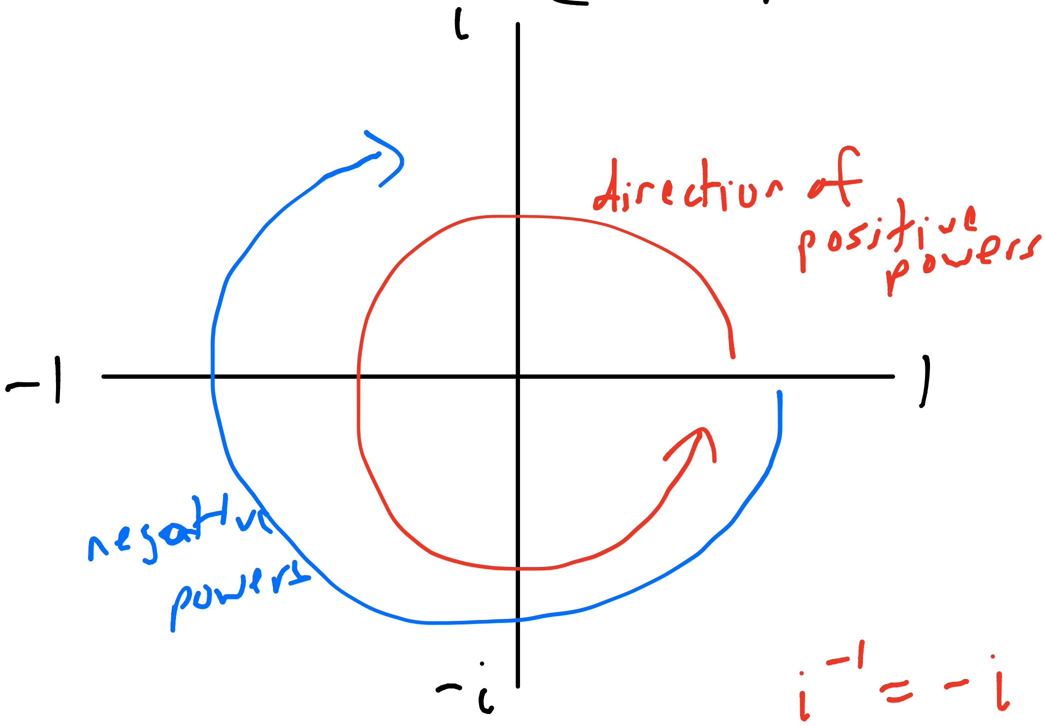
$i^{210} = i^{2} = -1$

$210 \div 4 = 52 \text{ R} 2$

- .0 → R0
- .25 → R1
- .5 → R2
- .75 → R3



$\mathbb{C}$ = complex numbers



$\mathbb{R}$ = real numbers

$i^{-1} = -i$

$i^{-2} = -1$

$i^{-3} = i$

$i^{-4} = 1$

$i^{18} = i^{2} = -1$

$\rightarrow 18 \div 4 = 4R2$

$i^{32} = i^{0} = 1$

$\rightarrow 32 \div 4 = 8R0$

$i^{107} = i^{3} = -1$

$\rightarrow 107 \div 4 = 26R3$

Complex Numbers are of the form $a+bi$

$a, b \in \mathbb{R}$, $i = \sqrt{-1}$

a = real component

b = complex / imaginary component

* Note: if $b=0$, $a+bi = a \in \mathbb{R}$ ← real number

$b \neq 0$, $a+bi \in \mathbb{C}$

↑ is a complex number

$$(a-bi)(a+bi) = a^2 - (bi)^2$$

difference of squares

$$= a^2 - b^2 i^2$$

$$= a^2 - b^2(-1)$$

$$(a-bi)(a+bi) = a^2 + b^2$$ ← sum of squares

x^2+9 was unfactorable before.

$$x^2+9 = (x+3i)(x-3i)$$
$$= (x)^2-(3i)^2$$
$$= x^2-3^2i^2$$
$$= x^2-9i^2$$
$$= x^2-9(-1)$$
$$= x^2+9$$

$$x^2+100 = (x)^2+(10)^2$$
$$= (x+10i)(x-10i)$$

* $a+b$ and $a-b$ are conjugates

b/c $(a+b)(a-b)=a^2-b^2$

→ $a+bi$ and $a-bi$ are complex conjugates

$(a+bi)(a-bi)=a^2+b^2$

$$(10-5i)(2+3i) = 20+15+30i-10i$$

$$= 35+20i$$

	2	$+3i$
10	20	$30i$
$-5i$	$-10i$	$-15i^2$ $+15$

Multiplying complex numbers, we expect to get a complex number???

$$(8-3i)(8+3i) = 64+9+24i-24i$$

$$= 73.$$

	8	$+3i$
8	64	$+24i$
$-3i$	$-24i$	$-9i^2$ $+9$

* Multiplying complex conjugates should result in a rational number.

$$\frac{4-3i}{5+2i} = \frac{4-3\sqrt{-1}}{5+2\sqrt{-1}} \cdot \frac{5-2\sqrt{-1}}{5-2\sqrt{-1}}$$

$$\hookrightarrow = \left(\frac{4-3i}{5+2i}\right)\cdot\left(\frac{5-2i}{5-2i}\right)$$

$$= \frac{(4-3i)(5-2i)}{(5+2i)(5-2i)} \leftarrow a^2+b^2$$

$$= \frac{14-23i}{(5)^2+(2)^2}$$

$$\begin{array}{c|c|c} & 4 & -3i \\ \hline 5 & 20 & -15i \\ \hline -2i & -8i & -6 \end{array}$$

$$= \frac{14-23i}{25+4}$$

$$= \frac{14-23i}{29}$$

$$= \frac{14}{29} - \frac{23}{29}i$$