Recall

$$\frac{5x}{3} + \frac{2}{3} = \frac{5x+2}{3}$$

* need common denominator to add/subtract

$$\frac{3}{7} - \frac{d}{7} = \frac{3-d}{7}$$

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* Let $a, b, c \in \mathbb{R}$ ← Let a, b, c be real numbers

$c \neq 0$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

We should remember that equality is commutative

$$a \cdot b = b \cdot a \qquad a+b = b+a$$

$$a = b \Leftrightarrow b = a$$

→ * $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$, $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

Division by monomial

$$\frac{6x^5 - 15x^3 + 9x^2 - 3x}{3x} = \frac{6x^5}{3x} - \frac{15x^3}{3x} + \frac{9x^2}{3x} - \frac{3x}{3x}$$

$$= 2x^4 - 5x^2 + 3x - 1$$

$$\frac{10c^3d - 15c^2d^2 + 2cd^3}{-5c^2d^2} = \frac{10c^3d}{-5c^2d^2} - \frac{15c^2d^2}{-5c^2d^2} + \frac{2cd^3}{-5c^2d^2}$$

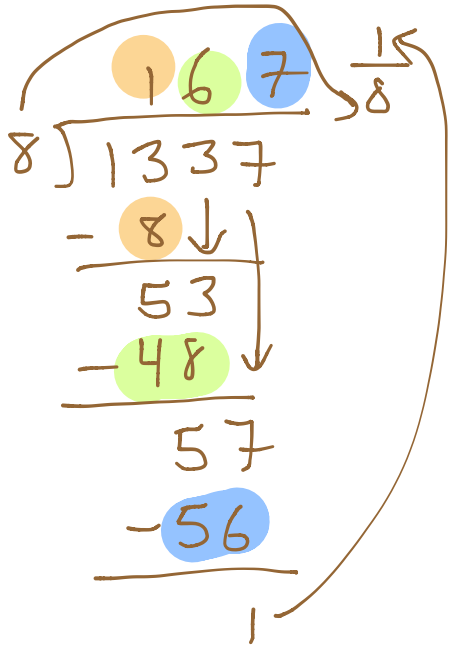
$$+ \frac{2}{-5}c^{1-2}d^{3-2}$$

$$+ \frac{2}{-5}c^{-1}d$$

* Note: answer should have positive powers!

$$= \frac{10c}{-5d} + \frac{15}{5} + \frac{2d}{-5c}$$

$$= -\frac{2c}{d} + 3 - \frac{2d}{5c}$$



$$\frac{1337}{8} = 167\frac{1}{8}$$

1 ← remainder

8 ← divisor

Dividing by Polynomials

$$3x - 8 + \frac{-26}{x-2}$$

$$x-2 \;\overline{)\; 3x^2 - 14x - 10}$$

$$-(3x^2 - 6x) \quad \downarrow$$

$$-8x - 10$$

$$-(-8x + 16)$$

$$-26$$

1. $\frac{3x^2}{x} = 3x$

2. $3x(x-2) = 3x^2 - 6x$

3. $3x^2 - 14x$
$-(3x^2 - 6x)$

*KCC $3x^2 - 14x$
$+ \; -3x^2 + 6x$
$-8x$

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1. $\frac{-8x}{x} = -8$

2. $-8(x-2) = -8x + 16$

3. $-8x - 10$
$-(-8x + 16)$

$-8x - 10$
$+ \; +8x - 16$
-26

$$\frac{-2x^3-10x^2+56}{2x-4}$$

$$\begin{array}{r} -x^2-7x-14 \\ 2x-4\overline{)\,-2x^3-10x^2+0x+56} \\ +(+2x^3\mp 4x^2)\ \downarrow \\ \hline -14x^2+0x \\ +(+14x^2\mp 28x)\ \downarrow \\ \hline -28x+56 \\ -(-28x+56) \\ \hline 0 \end{array}$$

remainder = 0

$\frac{-2x^3}{2x} = -x^2$

$-x^2(2x-4) = -2x^3+4x^2$

$\frac{-14x^2}{2x} = -7x$

$-7x(2x-4) = -14x^2+28x$

$\frac{-28x}{2x} = -14$

$-14(2x-4) = -28x+56$

$$\frac{-2x^3-10x^2+56}{2x-4} = -x^2-7x-14$$

$\rightarrow (2x-4)(-x^2-7x-14) = -2x^3+10x^2+56$

$2x-4$ and $-x^2-7x-14$ are factors of $-2x^3+10x^2+56$

*by synthetic division

$$\frac{3x^2-14x-10}{x-2}$$

$x-2=0$

$+2 \quad +2$

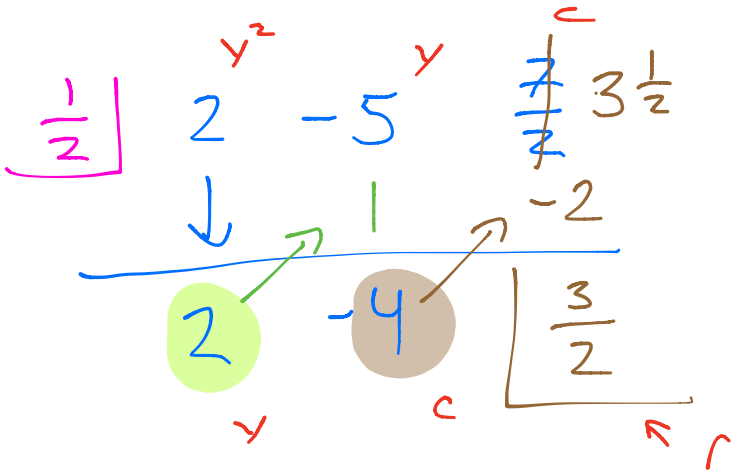
$x=2$

	x^2	x	c
2	3	−14	−10
		6	−16
	3	−8	−26
	x	c	remainder

$$3x-8-\frac{26}{x-2}$$

$$\frac{4y^2-10y+7}{2y-1}$$

$$2y-1=0$$
$$+1\ +1$$
$$2y=1$$
$$y=\frac{1}{2}$$



$$2y-4+\frac{\frac{3}{2}}{y-\frac{1}{2}}$$

$$2y-4+\frac{3}{2y-1}$$

Factoring

Identify the GCF of $9x^4 + 18x^3 - 6x^2$ $= 3x^2$

$9x^4 = 3^2 x^4$

$18x^3 = 2 \cdot 3^2 x^3$

$6x^2 = 2 \cdot 3 x^2$

notice all prime factors 3x shared and highest shared power x^2

all terms have factor of $3x^2$

$9 = 3^2$ (3)(3)

$18 = 2 \cdot 3^2$ (2)(9) (3)(3)

$6 = 1 \cdot 2$ (3)(2)

$$9x^4 + 18x^3 - 6x^2 = 3x^2\left(\frac{9x^4 + 18x^3 - 6x^2}{3x^2}\right)$$

$$9x^4 + 18x^3 - 6x^2 = 3x^2\left(\frac{9x^4}{3x^2} + \frac{18x^3}{3x^2} - \frac{6x^2}{3x^2}\right)$$

$$9x^4 + 18x^3 - 6x^2 = 3x^2(3x^2 + 6x - 2)$$

factored form

check by multiplying

Recall $(3a+2)(2b-7) = (3a+2)(2b) + (3a+2)(-7)$

$= 6ab + 4b - 21a - 14$

Factor by grouping

$6ab + 4b - 21a - 14$

$(6ab + 4b) + (-21a - 14)$

$2b(3a+2) + (-7)(3a+2)$

They are like terms now

$= (3a+2)(2b-7)$

GCF: $2b$

GCF: -7

* Let $3a+2 = u$

$2bu - 7u$

$u(2b-7)$

*GCF is $(3a+2)$

$x^3 + 3x^2 - 3x - 9$

Factor completely

$24p^2q^2 - 18p^2q + 60pq^2 - 45pq$

$= 3pq(8pq - 6p + 20q - 15)$

$= 3pq((8pq - 6p) + (20q - 15))$

$= 3pq(2p(4q - 3) + 5(4q - 3))$

$= 3pq(4q - 3)(2p + 5)$