Rational Expressions

$$\frac{6x^3}{5} \cdot \frac{2}{3x}$$

* Cross multiply is $\frac{a}{b} = \frac{c}{d}$

$ad = bc$

not for multiplying fractions

$$\frac{6x^3}{5} \cdot \frac{2}{3x} \longrightarrow \frac{6 \cdot 2x^3}{5 \cdot 3x} = \frac{\cancel{12}^{4}}{\cancel{15}_{5}} x^2 = \frac{4x^2}{5}$$

"multiplying across"

* order doesn't matter when multiplying "commutative property"

$$\frac{6x^3}{5} \cdot \frac{2}{3x} = \frac{6x^3}{3x} \cdot \frac{2}{5} = \frac{2x^2}{1} \cdot \frac{2}{5} = \frac{4x^2}{5}$$

* we can get away with "cross-canceling"..

$$\frac{\cancel{6x^3}^{2x^2}}{5} \cdot \frac{2}{\cancel{3x}_{1}} = \frac{4x^2}{5}$$

b/c $\frac{6x^3}{3x} = \frac{2x^2}{1}$

$$\frac{2x^4}{7} \div \frac{5x^4}{4}$$

$$= \frac{2x^4}{7} \cdot \frac{4}{5x^4}$$

"Keep Change Flip"

$$= \frac{2x^4}{7} \cdot \frac{4}{5x^4}$$

$$= \frac{8}{35}$$

$$\frac{15a^4}{14a^5b^1} \cdot \frac{21b^5}{25ab^1}$$

$$= \frac{3\cancel{15} \cdot \cancel{21}^{3} a^4 b^5}{\cancel{25}_{5} \cdot \cancel{14}_{2} a^6 b^2}$$

$$\frac{15}{25} = \frac{3}{5} \qquad \frac{21}{14} = \frac{3}{2}$$

$$= \frac{3 \cdot 3\, b^3}{5 \cdot 2\, a^2}$$

$$= \frac{9b^3}{10a^2}$$

$$\frac{x^2-64}{\cancel{3+x}} \cdot \frac{\cancel{x+3}}{x^2-8x}$$

$$= \frac{x^2-64}{x^2-8x}$$

$$= \frac{\cancel{(x-8)}(x+8)}{x\ \cancel{(x-8)}}$$

$$= \frac{x+8}{x}$$

$$\frac{x^2-36}{4x-x^2} \div \frac{3x-18}{x^2-16}$$

$$\frac{x^2-36}{4x-x^2} \cdot \frac{x^2-16}{3x-18}$$

$$\frac{(x-6)(x+6)}{x(4-x)} \cdot \frac{-1(x-4)(x+4)}{3(x-6)}$$

* Question $x-4$ and $4-x$?

try $x=1$

$1-4=-3$

$4-1=3$

try $x=2$

$2-4=-2$

$4-2=2$

$-1(b-a) = a-b$

$-b+a = a-b$

$a-b = a-b$ ✓

$$\frac{a-b}{b-a} = -1$$

$$\hookrightarrow \frac{x-4}{4-x} = -1$$

$$\hookrightarrow \frac{4-x}{x-4} = -1$$

$$\frac{(x+6)}{x(4-x)} \cdot \frac{-1(x-4)(x+4)}{3}$$

$$= \frac{-1(x+6)(x+4)}{3x}$$

$$= -\frac{(x+6)(x+4)}{3x}$$

Recall adding / subtracting fractions

$$\frac{a}{5} + \frac{b}{5} = \frac{a+b}{5}$$ * common denominator

$$\frac{a}{5} - \frac{b}{5} = \frac{a-b}{5}$$

$$\frac{3x+11}{x+2} + \frac{x-3}{x+2}$$

$$= \frac{(3x+x)+(11-3)}{x+2}$$

$$= \frac{4x+8}{x+2}$$

$$= \frac{4(x+2)}{(x+2)}$$

$$= 4$$

$$\frac{7r-1}{r^2} - \frac{6r-5}{r^2}$$

$$= \frac{(7r-1)-(6r-5)}{r^2}$$

$$= \frac{7r-1-6r+5}{r^2}$$

$$= \frac{r+4}{r^2}$$

$$\frac{7}{5b^4} - \frac{6}{4b^2}$$

denominators are not the same.
↳ need common denominator
prefer Least common denominator.

$5^1 b^4$ and $4b^2 = 2^2 b^2$

LCD: product of all prime factors to their greatest power

$5 \cdot 2 \cdot b$ ← prime factors

$5^1 \cdot 2^2 \cdot b^4$

LCD: $5 \cdot 2^2 \cdot b^4 =$

$5 \cdot 4\, b^4 = 20b^4$

$$\left(\frac{LCD}{LCD}\right)\frac{7}{5b^4} - \frac{6}{4b^2}\left(\frac{LCD}{LCD}\right)$$

$$\left(\frac{20b^4}{20b^4}\right)\frac{7}{5b^4} - \frac{6}{4b^2}\left(\frac{20b^4}{20b^4}\right)$$

$$\left(\frac{4}{20b^4}\right)\frac{7}{1} - \frac{6}{1}\left(\frac{5b^2}{20b^4}\right)$$

$$\frac{28 - 30b^2}{20b^4}$$

$$= \frac{2(14-15b^2)}{2(10b^4)}$$

$$= \frac{14-15b^2}{10b^4}$$

$$\frac{x+1}{x^2-6x+8} - \frac{3}{x^2-16}$$

$$= \frac{x+1}{(x-4)(x-2)} - \frac{3}{(x+4)(x-4)}$$

$1x^2 - 6x + 8$

* $a = 1$
  $b = -6$
  $c = 8$

$b = -6 = -4 + -2$

$c = 8 = -4 \cdot -2$

LCD:

$(x-4)(x-2)(x+4)$

↑ in both denominators but largest power is)

$$\frac{(x-4)(x-2)(x+4)}{(x-4)(x-2)(x+4)} \cdot \frac{x+1}{(x-4)(x-2)} - \frac{3}{(x+4)(x-4)} \cdot \frac{(x-4)(x-2)(x+4)}{(x-4)(x-2)(x+4)}$$

$$\frac{(x+4)(x+1) - 3(x-2)}{(x-4)(x-2)(x+4)}$$

$$= \frac{x^2+5x+4 \;-\; 3x+6}{(x-4)(x-2)(x+4)}$$

$$= \frac{x^2+2x+10}{(x-4)(x-2)(x+4)}$$ ← not factorable

$$\frac{\frac{5x^2}{y}}{\frac{10x}{y^2}} = \frac{5x^2}{y} \div \frac{10x}{y^2}$$

$$= \frac{5x^{\cancel{2}}}{\cancel{y}} \cdot \frac{\cancel{y^2}}{\cancel{10x}}$$

$$= \frac{xy}{2}$$