Factoring Quadratic Trinomials

(Multiplying)

Recall:

$(2x+3)(x+2) = 2x^2 + 4x + 3x + 6$

$= 2x^2 + 7x + 6$

	x	2
$2x$	$2x^2$	$4x$
3	$3x$	6

Factoring

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$ ($7x$ = $4x + 3x$)

$= (2x^2 + 4x) + (3x + 6)$

$= 2x(x+2) + 3(x+2)$

$= (2x+3)(x+2)$

Factor by Grouping

*

$7 = 4 + 3$

$4 \cdot 3 = 12 = 2 \cdot 6$

* Given $ax^2 + bx + c$ ← quadratic trinomial

$a, b, c \in \mathbb{R}$

$a \neq 0$

Need to find two values

whose sum is b

whose product is ac

ax^2+bx+c

$a=12$

$b=-5$

$c=-2$

$12x^2-5x-2$

$=12x^2+(-8+3)x-2$

$=12x^2-8x+3x-2$

$=(12x^2-8x)+(3x-2)$

$=4x(3x-2)+1(3x-2)$

$=(4x+1)(3x-2)$

$b=-5 = -8+3$

$ac=(12)(-2)=-24$ $\quad -8, 3$

-24

$1 \quad -24$

$-1 \quad 24$

$-2 \quad 12$

$2 \quad -12$

$-3 \quad 8$

$3 \quad -8 = -5$

$-4 \quad 6$

$4 \quad -6$

$10x^2+x-3 \quad =(2x-1)(5x+3)$ ✓

$b=1 = 6+-5$

$ac=-30 = 6 \cdot -5$

$10x^2+6x-5x-3$

$(10x^2+6x)+(-5x-3)$

$2x(5x+3)+(-1)(5x+3)$

$(2x-1)(5x+3)$

$-20c^3 + 34c^2d - 6cd^2$

$-2c(10c^2 - 17cd + 3d^2)$

$a = 10$

$b = -17d$

$c = 3d^2$

* c is our variable

$-2c(10c^2 - 15cd - 2cd + 3d^2)$

$ac = 30d^2 = \underline{-15d} \cdot \underline{-2d}$

$b = -17d = \underline{-15d} + \underline{-2d}$

$-2c((10c^2 - 15cd) + (-2cd + 3d^2))$

$-2c(5c(2c - 3d) + (-d)(2c - 3d))$

$-2c(5c - d)(2c - 3d)$

Let $a = 1$ so "$x^2 + bx + c$"

$z^2 - 10z + 16$

$z^2 - 8z - 2z + 16$

$(z^2 - 8z) + (-2z + 16)$

$z(z - 8) + (-2)(z - 8)$

$(z - 2)(z - 8)$

$a = 1$

$b = -10$

$z = 16$

$ac = 16 = \underline{-8} \cdot \underline{-2}$

$b = -10 = \underline{-8} + \underline{-2}$

$z^2 - 10z + 16 = (z - 2)(z - 8)$

$a^2 + 6a - 7$

$a^2 + 7a - 1a - 7$

$(a^2 + 7a) + (-a - 7)$

$a(a+7) - 1(a+7)$

$(a-1)(a+7)$

$a = 1$

$b = 6$

$c = -7$

$ac = -7 = \underline{-1} \cdot \underline{7}$

$b = 6 = \underline{-1} + \underline{7}$

Given $x^2 + bx + c \rightarrow$ Let $b = m + n \rightarrow (x+m)(x+n)$

$m - n \rightarrow (x+m)(x-n)$

$-m - n \rightarrow (x-m)(x-n)$

$-m + n \rightarrow (x-m)(x+n)$

$x^2 + 15x + 50$

$= (x+10)(x+5)$

* $a = 1$

$b = 15$

$c = 50$

$b = \underline{10} + \underline{5}$

$c = \underline{10} \cdot \underline{5}$

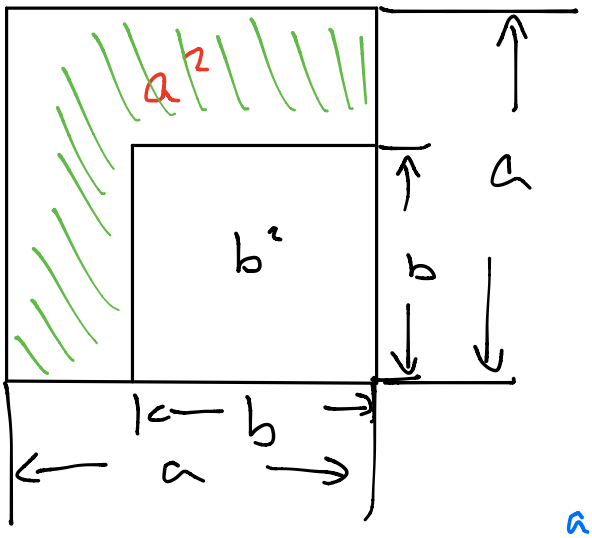
Check

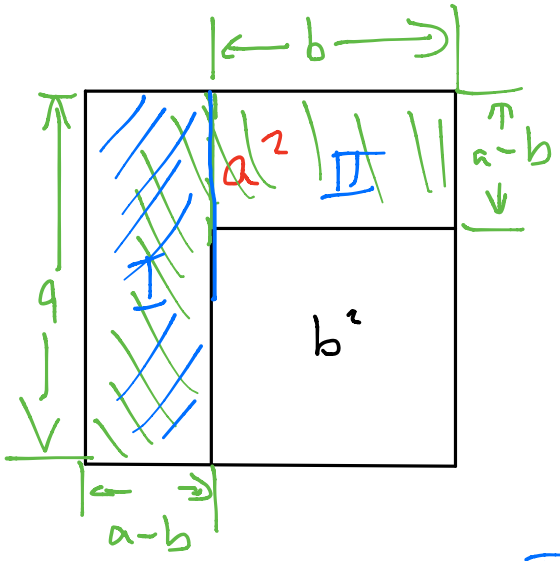
$(x+10)(x+5) = x^2 + 10x + 5x + 50$

$= x^2 + 15x + 50$

	x	5
x	x^2	$5x$
10	$10x$	50

Consider $a^2 - b^2$





$a^2 + b^2 =$ I + II

$$a^2 - b^2 = a(a-b) + b(a-b)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$x^2 - 4 = (x)^2 - (2)^2$$

Difference of Squares

$$x^2 - 4 = (x+2)(x-2)$$

$$16x^2 - 9 = (4x)^2 - (3)^2$$

$$= (4x+3)(4x-3)$$

$$4z^2 - 1 = (2z)^2 - (1)^2$$

$$= (2z+1)(2z-1)$$

$$z^2 - 49a^6 = (z)^2 - (7a^3)^2$$

$$= (z - 7a^3)(z + 7a^3)$$

$$z^4 - 81 = (z^2)^2 - (9)$$

$$= (z^2 - 9)(z^2 + 9)$$

$$= (z-3)(z+3)(z^2+9)$$

not factorable at this moment.