[MODULE 3: EQUATION OF A LINE PARALLEL AND PERPENDICULAR LINES]

New York City College of Technology
MAT 1175 PLTL Workshops

Name: _____ Points: _____

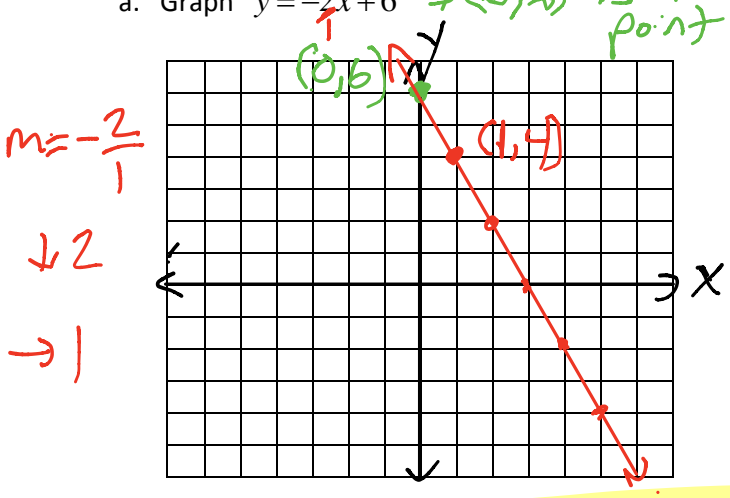
Given two points (x_1, y_1) and (x_2, y_2), the formula for finding the slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

Slope-Intercept Form of the Equation of a Line: The equation of any line with slope m and y-intercept b is given by $y = mx + b$. y-intercept $(0, b)$

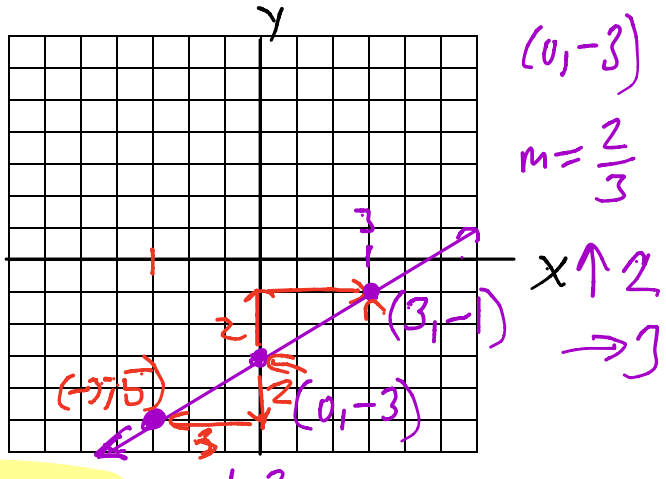
Point-Slope Form of the Equation of a Line: The equation of the line through (x_1, y_1) with slope m is given by $y = m(x - x_1) + y_1$.

1. Graph by the $y = mx + b$ method

a. Graph $y = -2x + 6$ → $(0, b)$ is a point

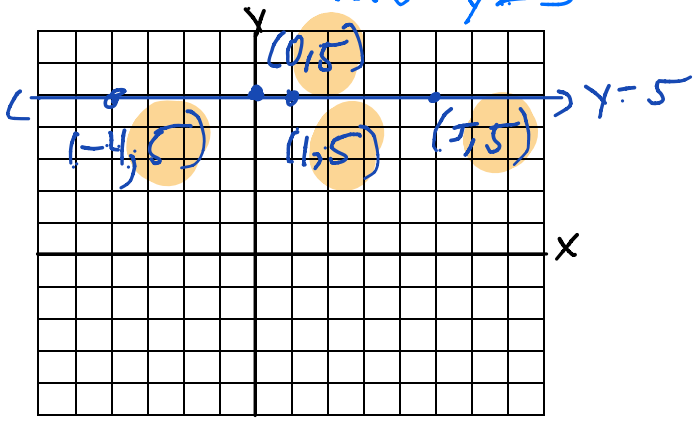


b. Solve the equation for y and graph: $2x - 3y = 9$

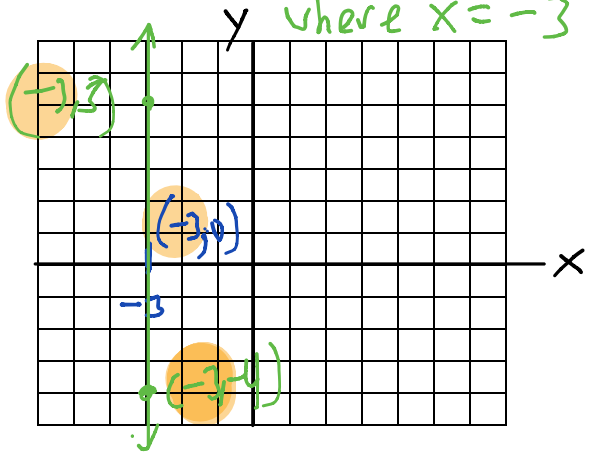


2. Special Lines: The graph of $y = a$ is a horizontal line through $(0, a)$
The graph of $x = a$ is a vertical line through $(a, 0)$

a. Graph $y = 5$ = set of all points where $y = 5$



b. Graph $x = -3$ → set of all points where $x = -3$



3. Find the equation of the line with slope $-\frac{2}{3}$ and y-intercept $(0,4)$.

$y = -\frac{2}{3}x + 4$

4. Give the slope and y-intercept for the line $x - 2y = 1$.

$y = mx + b$

$x - 2y = 1$

$-x \quad -x$

$-2y = -x + 1$

$* -x + 1 = 1 - x$

$\frac{-2y}{-2} = \frac{-x+1}{-2}$

$y = \frac{-1x}{-2} + \frac{1}{-2}$

$y = \frac{1}{2}x - \frac{1}{2}$

slope: $\frac{1}{2}$

y-int: $(0, -\frac{1}{2})$

5. Find the equation of the line that passes through the points $(-4,5)$ and $(-7,4)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{(4)-(5)}{(-7)+(+4)} = \frac{-1}{-3} = \frac{1}{3}$

point slope form

$y = m(x - x_1) + y_1$

$y = \frac{1}{3}(x - (-4)) + (5)$

$y = \frac{1}{3}(x+4) + 5\left(\frac{3}{3}\right)$

$y = \frac{1}{3}x + \frac{4}{3} + \frac{15}{3}$

$y = \frac{1}{3}x + \frac{19}{3}$

Parallel Lines: Non vertical parallel lines have the same slope.

Perpendicular Lines: The slopes of perpendicular lines are negative reciprocals.

6. a. Write an equation of a line passing through the point $(-2,1)$ and perpendicular to $y = \frac{2}{3}x - 10$

$m_{\perp} = -\frac{3}{2}$

$y = m(x - x_1) + y_1$

$y = -\frac{3}{2}(x - (-2)) + (1)$

$y = -\frac{3}{2}(x+2) + 1$

$y = -\frac{3}{2}x - 3 + 1$

$y = -\frac{3}{2}x - 2$

b. Write an equation of a line passing through the point $(5,1)$ and parallel to $y = -\frac{2}{5}x + 1$

$m_{\parallel} = -\frac{2}{5}$

7. Write an equation of a line passing through the point $(-12,3)$ and

a. parallel to $4x - 3y = 7$

b. perpendicular to $4x - 3y = 7$

y-intercept – the point where the graph passes through the y-axis

slope $= \frac{\text{rise}}{\text{run}}$ ↑ →

- $(0, b)$
- b is the y-value when $x = 0$

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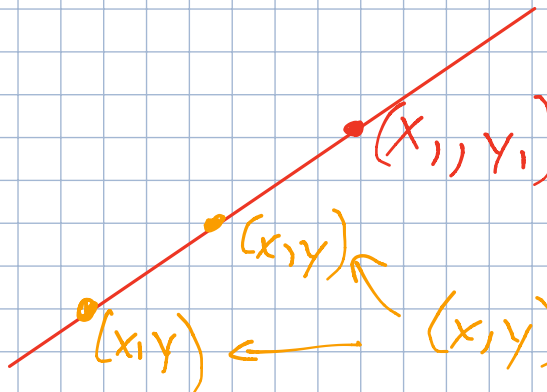
point slope form of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \cdots \quad m = \frac{y - y_1}{x - x_1}$$

(x, y) is any point on a line

(x_1, y_1) is a specific point on a line



(x, y) can be any point on the line except (x_1, y_1)

* We generally prefer to write line as equations as $y = mx + b$

$$m = \frac{y - y_1}{x - x_1}$$

Solve for y

$$m(x - x_1) = \frac{y - y_1}{x - x_1}\,(x - x_1)$$

*multiply or cross multiply

$$m(x - x_1) = y - y_1$$

$$y - y_1 = m(x - x_1)$$ → point slope form of a line

(x, y) is an arbitrary point.

(x_1, y_1) is a given point

m = slope

$$y - y_1 = m(x - x_1)$$

$$+y_1 \qquad +y_1$$

$$y = m(x - x_1) + y_1$$

Standard form of a line

$$Ax + By = C$$

(x, y) is any point on the line.

$2x - 3y = 9$

$-2x \qquad -2x$

$-3y = -2x + 9$

Put into $y = mx + b$ form and graph.

* get y by itself

* $-3y$ means -3 times y

$$\frac{-3y}{-3} = \frac{-2x+9}{-3}$$

$$y = \frac{-2x+9}{-3}$$

$$y = \frac{-2}{-3}x + \frac{9}{-3}$$

$$y = \frac{2}{3}x - 3$$

$(0, -3)$ ← y-int

$m = \text{slope} = \frac{2}{3}$

$$y = \frac{1}{3}(x - (-7)) + (4)$$

$$y = \frac{1}{3}(x + 7) + 4$$

$$y = \frac{1}{3}x + \frac{7}{3} + \frac{4}{1}\left(\frac{3}{3}\right)$$

$$y = \frac{1}{3}x + \frac{7}{3} + \frac{12}{3}$$

$$y = \frac{1}{3}x + \frac{19}{3}$$

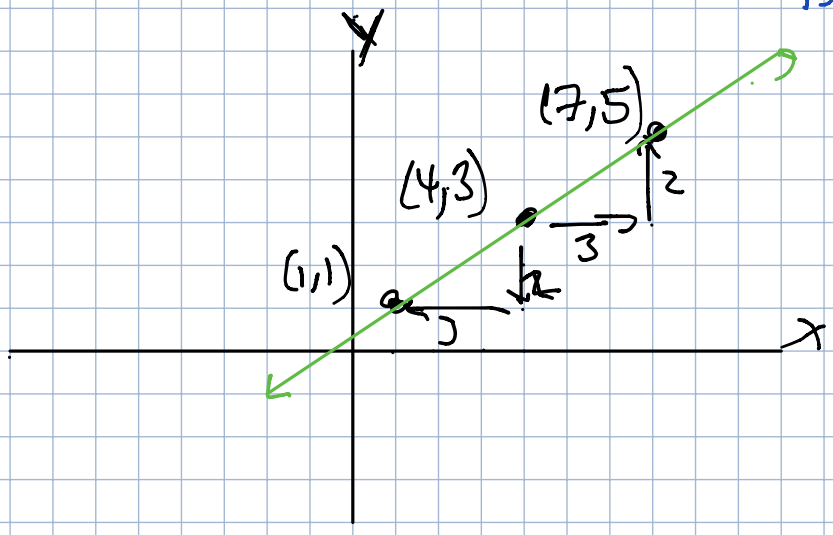
Graphing in point-slope

$y = m(x - x_1) + y_1$, (x_1, y_1) is point

$y = \frac{2}{3}(x - 4) + 3$

point: $(4, 3)$

slope: $\frac{2}{3}$ ↑ 2, → 3



1. Plot the point in the equation
2. Use slope to find second point
3. Draw line between 2 points

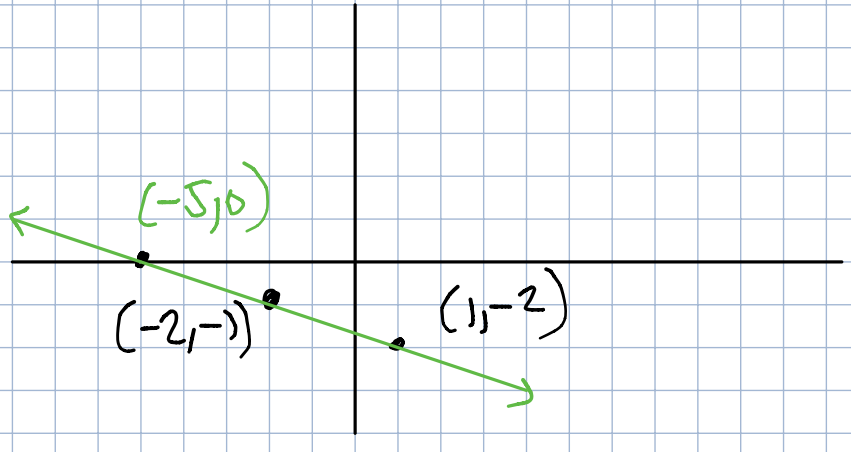
$y = -\frac{1}{3}(x + 2) - 1$

point: $(-2, -1)$

slope: $-\frac{1}{3}$ ↓ 1, → 3

* $y = m(x - x_1) + y_1$

$y = -\frac{1}{3}(x - (-2)) + (-1)$



* Parallel lines have same slope.
* if one line has slope $\frac{y}{x}$, a perpendicular line has a slope $-\frac{x}{y}$