A probability distribution that plots all of its values in a symmetrical fashion and most of the results are situated around the probability's mean is called a **normal distribution**. Values are equally likely to plot either above or below the mean. Grouping takes place at values that are close to the mean and then tails off symmetrically away from the mean.

Some Properties of a Normal Distribution

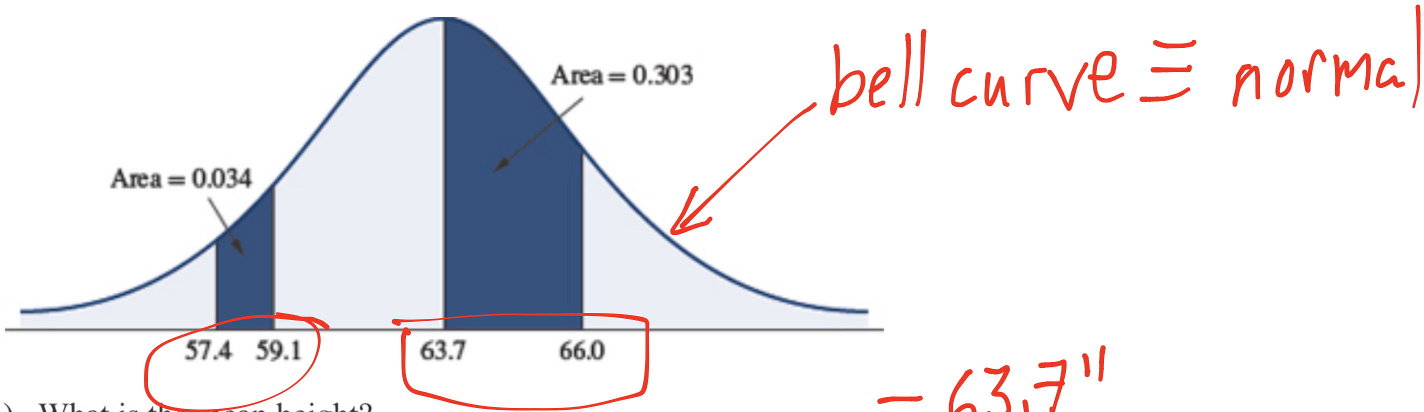
1. The value in the middle of the distribution, which appears most often in the sample, is the mean.
2. The distribution is symmetric about the mean. This means that the graph has two halves that are mirror images on either side of the mean value.
3. This is the key fact: the area under any portion of the curve is the percentage (in decimal form) of data values that fall between the values that begin and end that region.
4. The total area under the entire curve is 1.

Area = probability

→ total Area = max probability = 1

Example 1 Getting Information from a Normal Curve

The graph below shows a normal distribution for heights of women in the United States. The numbers on the horizontal axis are heights in inches, and some areas are labeled for reference.



(a) What is the mean height? $= 63.7''$

(b) What percentage of women are between 57.4 and 59.1 inches tall? $= 0.034 \approx 3.4\%$

(c) If there are 31,806 women at a stadium concert, how many of them would you expect to be between 63.7 and 66.0 inches tall?

c.) percentage : 30.3%

# of women between 63.7'' and 66''

$(0.303)(31806) \approx 9637.2$

≈ 9637 women

The Empirical Rule

When data are normally distributed, approximately 68% of the values are within 1 standard deviation of the mean, approximately 95% are within 2 standard deviations of the mean, and approximately 99.7% are within 3 standard deviations of the mean (see **Figure 11-10**).

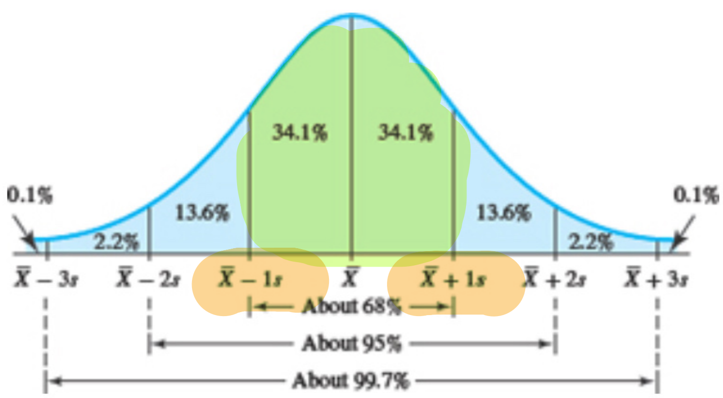


Figure 11-10 $\bar{X}$ = mean, s = standard deviation.

Example 2 Using the Empirical Rule

According to the website answerbag.com, the mean height for male humans is 5 feet 9.3 inches, with a standard deviation of 2.8 inches. If this is accurate, out of 1,000 randomly selected men, how many would you expect to be between 5 feet 6.5 inches and 6 feet 0.1 inch?

$\bar{x}$ = 5ft. 9.3 in

s = 2.8 in

1. Need to find how many standard deviations is 5ft 6.5 in

5ft 6.5 in − 5ft 9.3 in = −2.8 in

= −1 SD

$x - \bar{x}$

data value − mean

2. Need to find how many standard deviations is 6ft 0.1 in

6ft. 0.1 in − 5ft. 9.3 in =

5ft 12.1 in − 5ft 9.3 in = 2.8 in

* b/c 0.1 in is less than 9.3 inches borrow from feet & convert to inches

2.8 in = 1 sd

That means between

$\bar{x} + 1\,sd$ and $\bar{x} - 1\,sd$

68.2% of values in this interval

out of 1000 men, 682 are within 1SD

$(.682)(1000) = 682$

The **standard normal distribution** is a normal distribution with mean 0 and standard deviation 1.

The standard normal distribution is shown in **Figure 11-11**. The values under the curve shown in **Figure 11-11** indicate the proportion of area in each section. For example, the area between the mean and 1 standard deviation above or below the mean is about 0.341, or 34.1%.

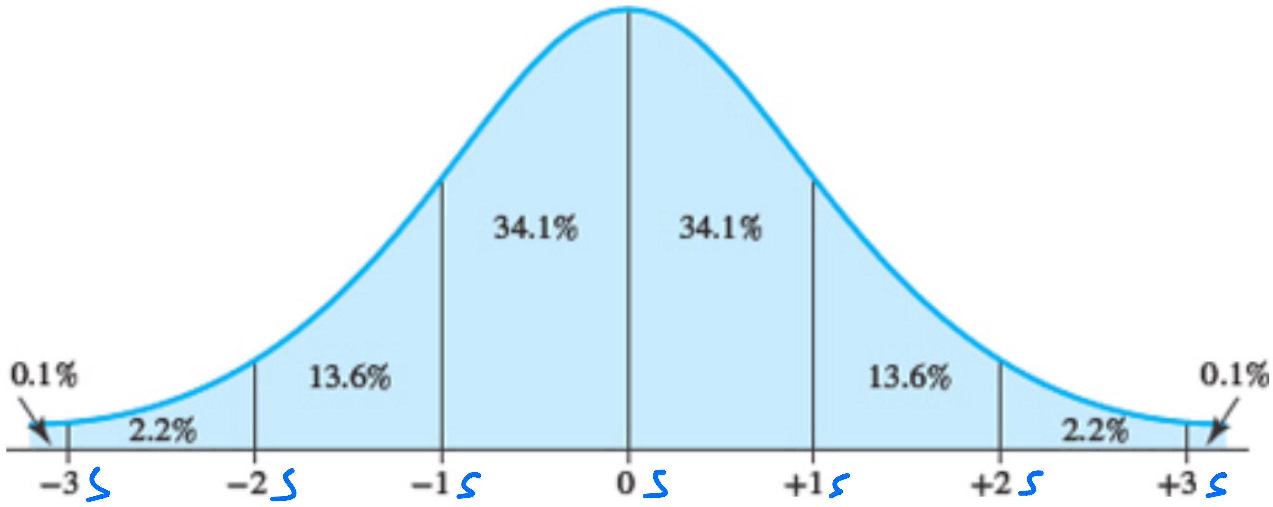


Figure 11-11

$\bar{x}$

For a data value from a sample with mean $\overline{X}$ and standard deviation *s* the *z* **score** is

$$z = \frac{X - \overline{X}}{s}$$

Verbally, to find a *z* score, just subtract the mean, then divide the result by the standard deviation.

Example 3 Computing a *z* Score

Based on the information in **Example 2**, find the *z* score for a man who is 6 feet 4 inches tall and describe what it tells us.

mean $\bar{x}$ = 5ft 9.3in = 69.3in

data value x = 6ft 4 in = 76 in

SD s = 2.8in

$$z = \frac{x - \bar{x}}{s} = \frac{(76)-(69.3)}{2.8} \approx +2.39$$

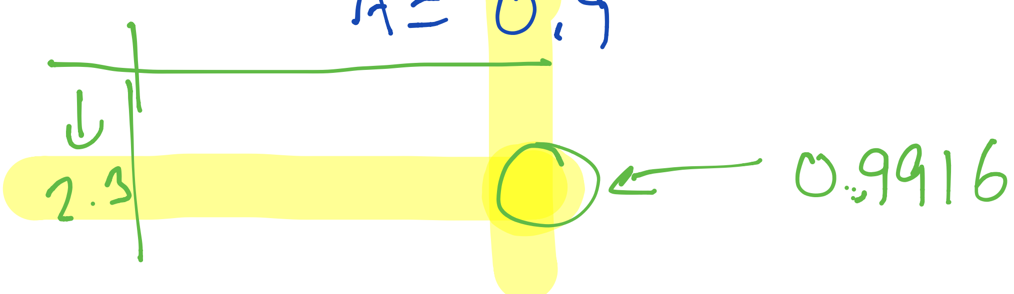
*The man who is 6ft 4in tall is taller than the average male

z-score measures number of standard deviations

use chart

+2.39 → look at cell for z = 2.39

A = 0.9



.9916 says that 6ft 4'in man is taller than 99.16% of all men.

Example 4 **Using *z* Scores to Compare Standardized Test Scores**

$z = \frac{x - \bar{x}}{s}$

As you probably know, there are two main companies that offer standardized college entrance exams, ACT and SAT. Since each has a completely different scoring scale, it's really difficult to compare the scores of students that took different exams. One year the ACT had a mean score of 21.2 and a standard deviation of 5.1. That same year, the SAT had a mean score of 1498 and a standard deviation of 347. Suppose that a scholarship committee is considering two students, one who scored 26 on the ACT and another who scored 1800 on the SAT. Both are pretty good scores, but which one is better?

ACT score: $z = \frac{26 - 21.2}{5.1} = 0.94$

SAT score: $z = \frac{1800 - 1498}{347} = 0.87$

ACT score better. Higher z-score.

Example 7	Finding the Area to the Left of a z Score

Find the area under the standard normal distribution

(a) To the left of $z = -2.20$. $= 0.0139$

(b) To the left of $z = 1.95$.

$= 0.9744$

use chart

	0.00
−2.2	

0.0139

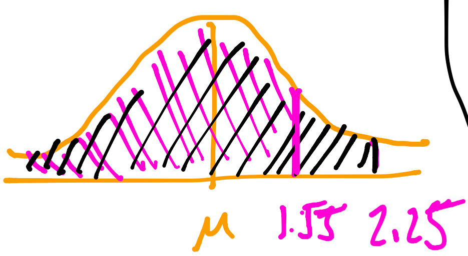
Example 5	Finding the Area between Two z Scores

Find the area under the standard normal distribution

(a) Between $z = 1.55$ and $z = 2.25$.

(b) Between $z = -0.60$ and $z = -1.35$.

(c) Between $z = 1.50$ and $z = -1.75$.



	0.00	0.01	…	0.05
2.2				.98778

	0.00
−.6	.27425

(a) $A_{z=2.25} - A_{z=1.55}$

$.98778 - .93943$

≈ 0.04835

(b) $A_{z=-0.60} = .27425$

$A_{z=-1.35} = .08851$

$.27425 - .08851 = .18574$

(c) $A_{z=1.50} = 0.93319$

$A_{z=-1.75} = 0.04006$

$0.93319 - .04006$

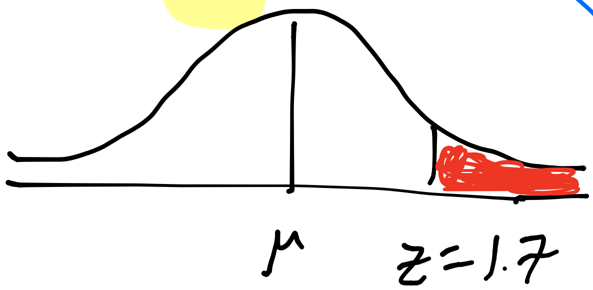
$= .89313$

Example 6 **Finding the Area to the Right of a z Score**

Find the area under the standard normal distribution

(a) To the right of $z = 1.70$.

(b) To the right of $z = -0.95$.



Recall that the area underneath normal curve $= 1$

$1 - A_{z=1.70}$

$= 1 - (0.95543)$

$= 0.04457$

$1 - A_{z=-0.95}$

$= 1 - 0.17106$

$= .82894$

Example 1 **Solving a Problem Using the Normal Distribution**

If the weights of Oreos in a package are normally distributed with mean 518 grams and standard deviation 4 grams, find the percentage of packages that will weigh less than 510 grams.

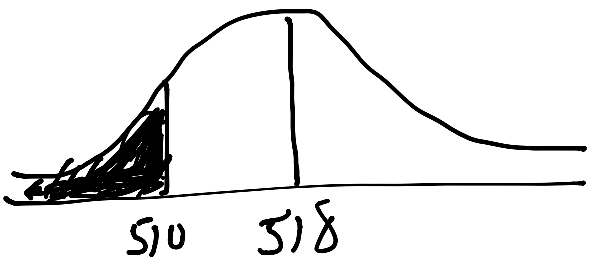
$$z = \frac{x - \bar{x}}{s}$$

z = z score

x = data value = 510

$\bar{x}$ = mean = 518

s = standard deviation = 4



$$z = \frac{510 - 518}{4} = -2 \rightarrow z = -2$$

$$A_{z=-2} = 0.02275$$

2.275%

Probability and Area under a Normal Distribution

The area under a normal distribution between two data values is the probability that a randomly selected data value is between those two values.

Example 2 illustrates the use of area to find probability.

Example 2 Using Area under a Normal Distribution to Find Probabilities

$\bar{x} = 1606$

$S = 200$

Based on data in an EPA study released in 2016, the average American generates 1,606 pounds of garbage per year. Let's estimate that the number of pounds generated per person is approximately normally distributed with standard deviation 200 pounds. Find the probability that a randomly selected person generates

(a) Between 1,250 and 2,050 pounds of garbage per year.

(b) More than 2,050 pounds of garbage per year.

a.) $x_1 = 1250$ $x_2 = 2050$

$z_1 = \frac{1250-1606}{200}$ $z_2 = \frac{2050-1606}{200}$

$z_1 = -1.78$ $z_2 = 2.22$

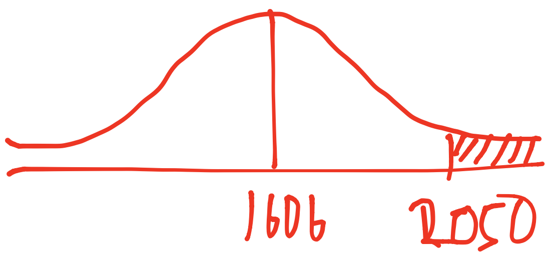
$\rightarrow A_{z=-1.78} = .03754$ $\rightarrow A_{z=2.22} = .98679$

b/c $z_2 > z_1$

$A_{z_2} - A_{z_1}$

$= .98679 - .03754 = .94925$

b.) More than 2050 lbs garbage per year



to the right

$$z = \frac{x-\bar{x}}{s} = \frac{2050-1606}{200}$$

$$= 2.22$$

$$A_{z=2.22} = .98679$$

$$1 - A_{z=2.22}$$

$$1 - .98679 = .01321$$