## Counting

Counting? You already know how to count or you wouldn't be taking a college-level math class, right? Well yes, but what we'll really be investigating here are ways of counting *efficiently*. When we get to the probability situations a bit later in this chapter we will need to count some *very* large numbers, like the number of possible winning lottery tickets. One way to do this would be to write down every possible set of numbers that might show up on a lottery ticket, but believe me: you don't want to do this.

Basic Counting

We will start, however, with some more reasonable sorts of counting problems in order to develop the ideas that we will soon need.

Example 21

Suppose at a particular restaurant you have three choices for an appetizer (soup, salad or breadsticks) and five choices for a main course (hamburger, sandwich, quiche, fajita or pizza). If you are allowed to choose exactly one item from each category for your meal, how many different meal options do you have?

3 appetizers

Soup

Salad

breadsticks

5 main

hamburger

sandwich

quiche
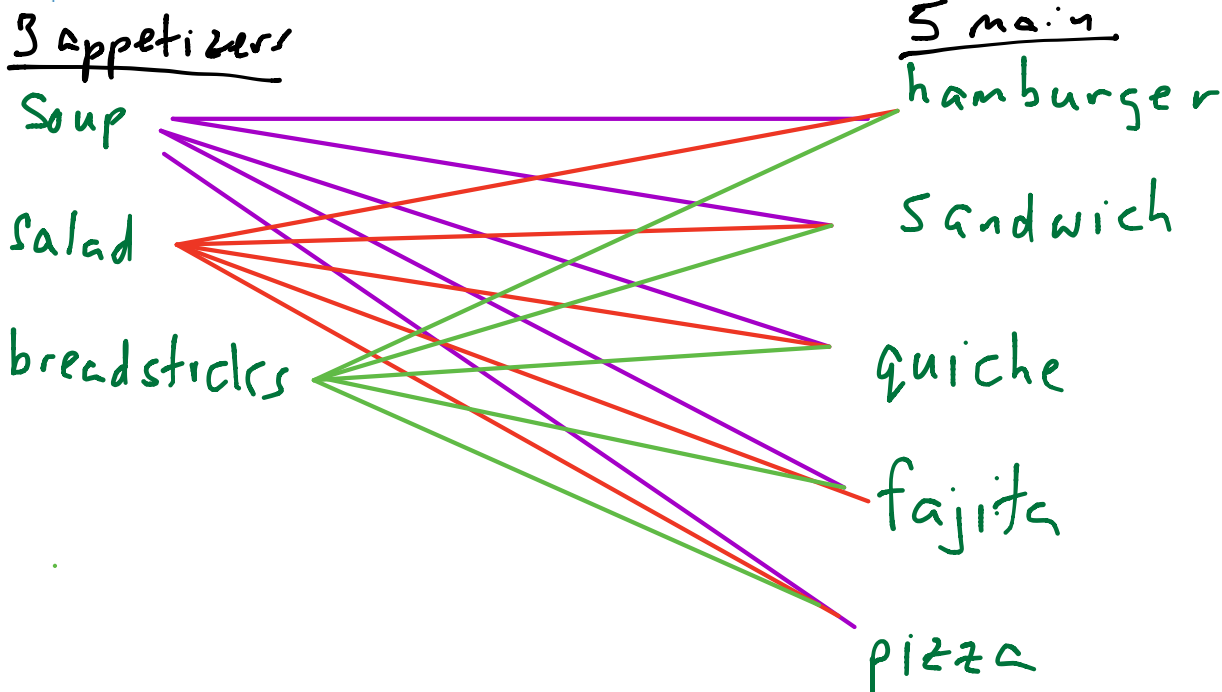
fajita

pizza

5 lines for soup

5 lines for salad

5 lines for breadsticks
_____

15 options for appetizers & main courses

$3 \times 5 = 15$ options

	hamburger	sandwich	quiche	fajita	pizza
Soup	S_o, H	S_o, S	S_o, Q	S_o, F	S_o, P
salad	S_a, H	S_a, S	S_a, Q	S_a, F	S_a, P
breadsticks	B, H	B, S	B, Q	B, F	B, P

each event is one cell

15 cells for 15 pairings

3 appetizers × 5 main courses = 15 pairings

Appetizers **Main**

Soup
- H → S_o, H
- S → S_o, S
- Q → S_o, Q
- F → S_o, F
- P → S_o, P

Salad
- H → S_a, H
- S → S_a, S
- Q → S_a, Q
- F → S_a, F
- P → S_a, P

Breadsticks
- H → B, H
- S → B, S
- Q → B, Q
- F → B, F
- P → B, P
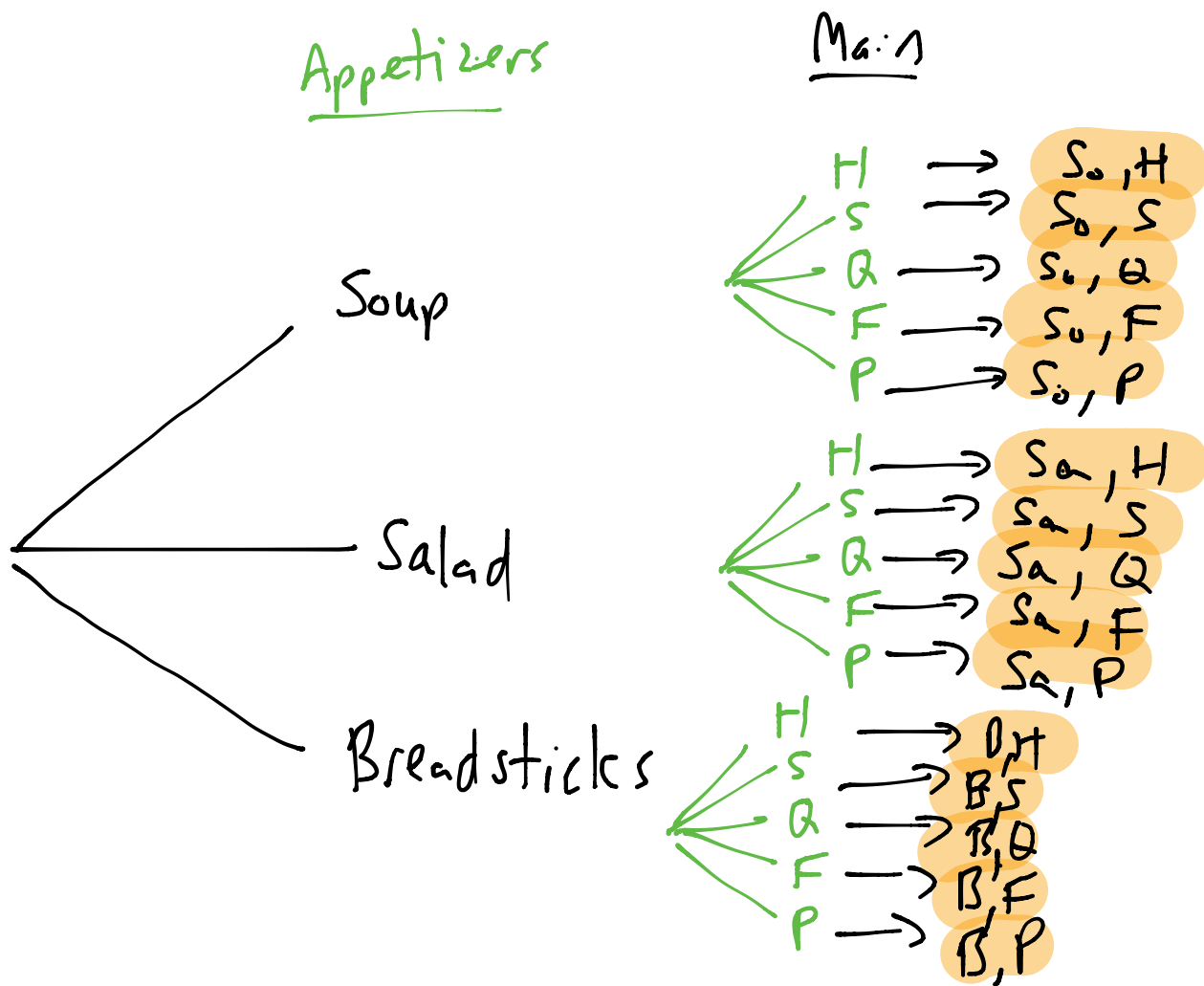
Generate Sample Space by following branches from the beginning to end

> **Basic Counting Rule**
> If we are asked to choose one item from each of two separate categories where there are m items in the first category and n items in the second category, then the total number of available choices is $m \cdot n$.
>
> This is sometimes called the multiplication rule for probabilities.

Example 22

> There are 21 novels and 18 volumes of poetry on a reading list for a college English course. How many different ways can a student select one novel and one volume of poetry to read during the quarter?

21 novels * 18 volumes = 378 combinations

378 possible outcomes in our sample space

Example 23

> Suppose at a particular restaurant you have three choices for an appetizer (soup, salad or breadsticks), five choices for a main course (hamburger, sandwich, quiche, fajita or pasta) and two choices for dessert (pie or ice cream). If you are allowed to choose exactly one item from each category for your meal, how many different meal options do you have?
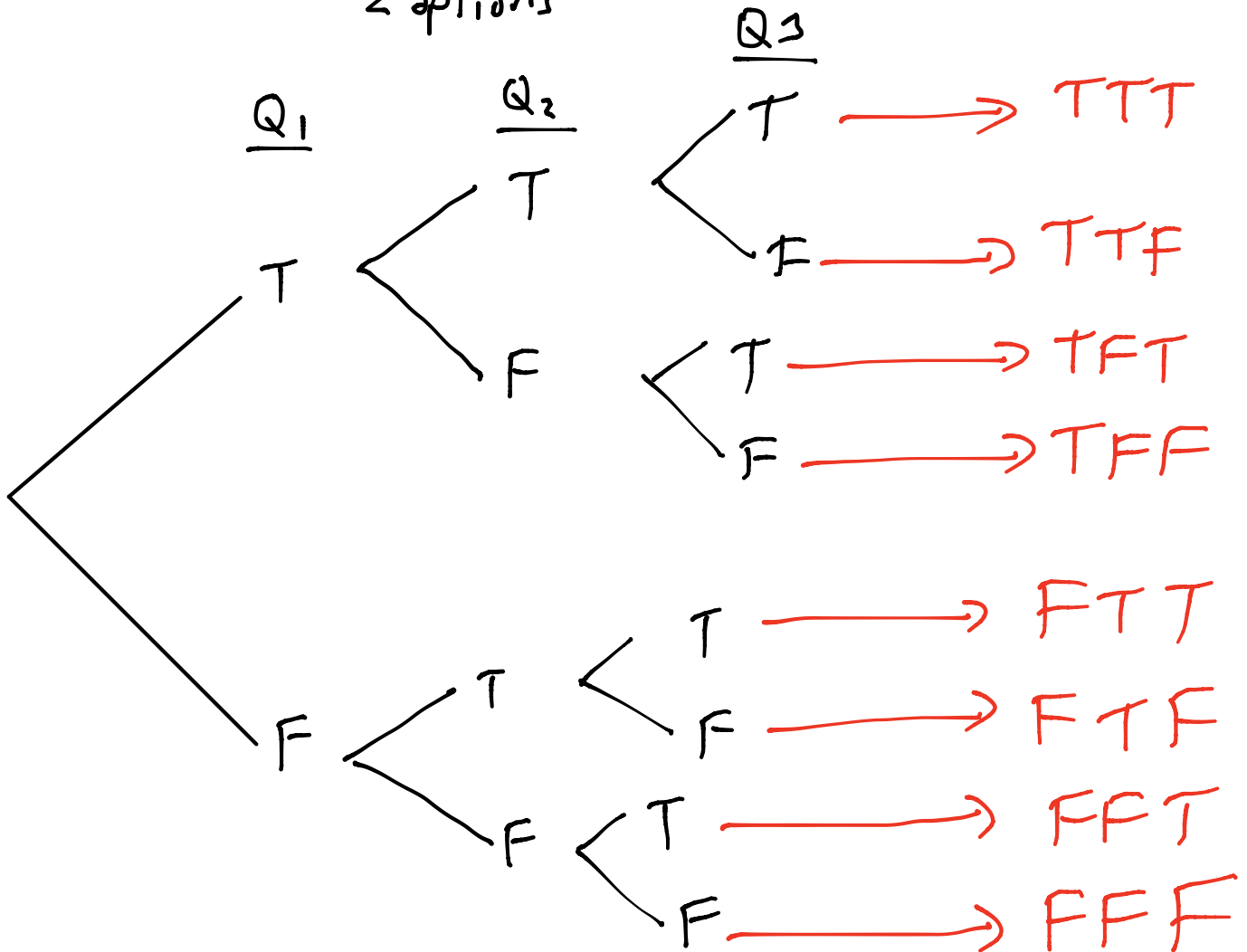
3 appetizers * 5 mains * 2 desserts = 30 options

Example 24

A quiz consists of 3 true-or-false questions. In how many ways can a student answer the quiz?

$3 \times 2 = ?$ No

2 options

Q_1 Q_2 Q_3

T → TTT

TTF

TFT

TFF

FTT

FTF

FFT

FFF

2 for Q_1 ✶ 2 for Q_2 ✶ 2 for Q_3

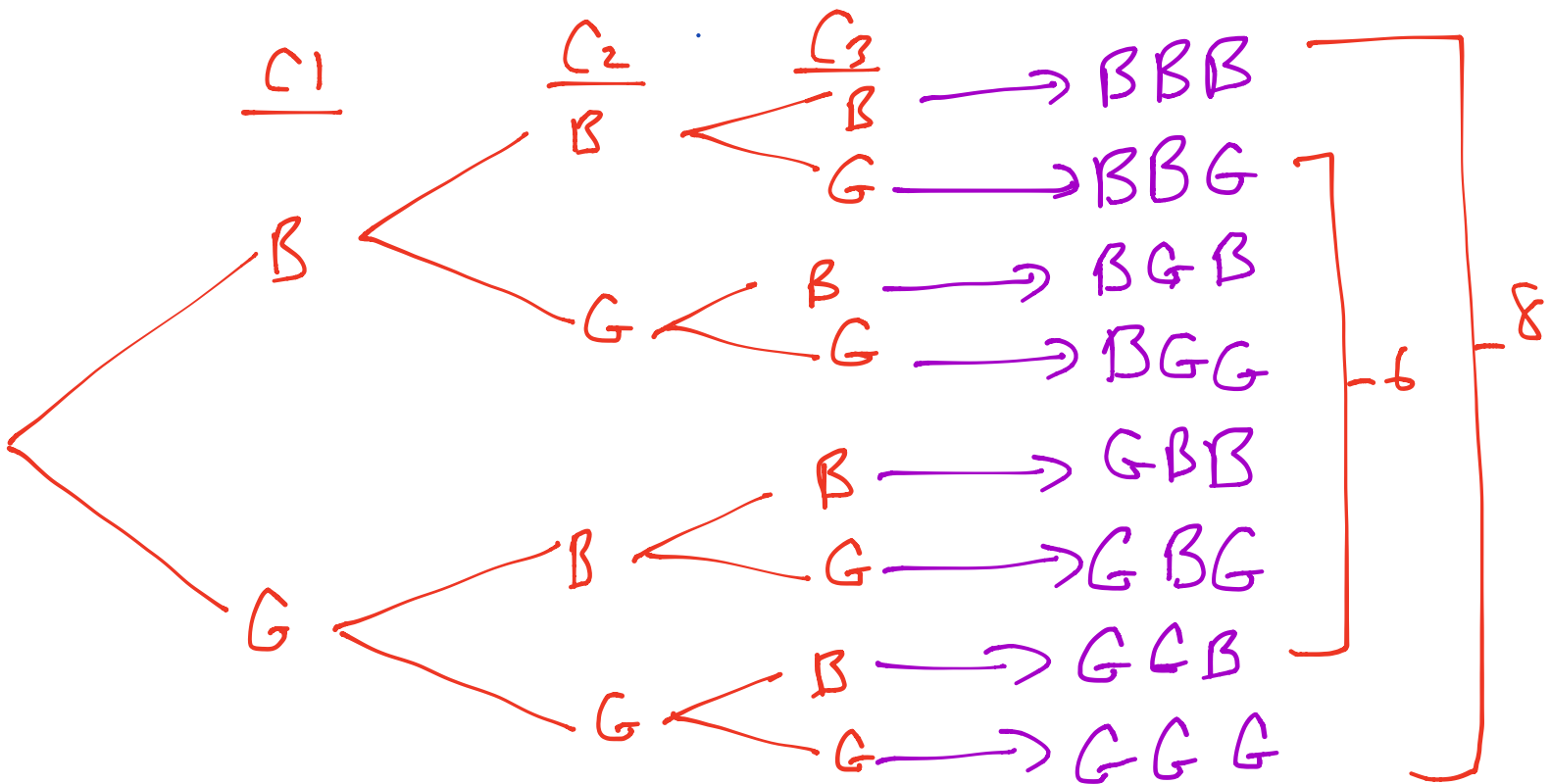$2 \times 2 \times 2 = 8$ possible ways of answering

$2^3 = 8$

Example 1 **Using a Tree Diagram to Find a Sample Space**

Use a tree diagram to find the sample space for the genders of three children in a family.

Example 2 **Computing a Probability**

If a family has three children, find the probability that they have at least one boy and one girl. (Assume that each child is equally likely to be a boy or girl.)

$$C1 \qquad \frac{C_2}{B} \qquad \frac{C_3}{B} \longrightarrow BBB$$

$$B < \begin{array}{c} B < \begin{array}{c} B \longrightarrow BBB \\ G \longrightarrow BBG \end{array} \\ G < \begin{array}{c} B \longrightarrow BGB \\ G \longrightarrow BGG \end{array} \end{array}$$

$$G < \begin{array}{c} B < \begin{array}{c} B \longrightarrow GBB \\ G \longrightarrow GBG \end{array} \\ G < \begin{array}{c} B \longrightarrow GGB \\ G \longrightarrow GGG \end{array} \end{array}$$

$$\frac{6 \text{ outcomes } (boy \& girl)}{8 \text{ total outcomes}} =$$

$$P(B \geq 1 \text{ and } G \geq 1) = \frac{6}{8} = \frac{3}{4}$$