Example 42

[1]In the casino game roulette, a wheel with 38 spaces (18 red, 18 black, and 2 green) is spun. In one possible bet, the player bets \$1 on a single number. If that number is spun on the wheel, then they receive \$36 (their original \$1 + \$35). Otherwise, they lose their \$1. On average, how much money should a player expect to win or lose if they play this game repeatedly?

$P(\text{winning}) = \frac{1}{38}$ Result: $+\$35$

$P(\text{loss}) = \frac{37}{38}$ Result: $-\$1$

Value of winning + Value of losing

$$(\$35)\left(\frac{1}{38}\right) + (-\$1)\left(\frac{37}{38}\right)$$

$$\frac{\$35}{38} - \frac{\$37}{38} = -\frac{\$2}{38}$$

$$\approx -\$0.052\ldots$$

$$\approx -\$0.05$$

→ the expected value is a loss of \$0.05 per play.

Note: the expected value is not the same as an outcome.
in this case, the outcomes are $-\$1$ or $+\$35$

Expected Value

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.

sum

probability of outcome

$$EV = \sum_{n=1}^{m} \left(x\, P(x) \right)$$

expected value

outcome

$$EV = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + \dots$$

Try it Now 12

You purchase a raffle ticket to help out a charity. The raffle ticket costs $5. The charity is selling 2000 tickets. One of them will be drawn and the person holding the ticket will be given a prize worth $4000. Compute the expected value for this raffle.

	Outcome	Probability	$x \cdot Px$
winning	$4000 - 5 = 3995$	$\frac{1}{2000}$	$3995\left(\frac{1}{2000}\right) = \frac{3995}{2000}$
losing	$0 - 5 = -5$	$\frac{1999}{2000}$	$-5\left(\frac{1999}{2000}\right) = \frac{-9995}{2000}$

$$EV = \sum x P(x)$$

$$= \frac{3995}{2000} + -\frac{9995}{2000} = -\frac{6000}{2000} = -\$3$$

expect $\boxed{\$3 \quad \text{loss}}$

Example 43

In a certain state's lottery, 48 balls numbered 1 through 48 are placed in a machine and six of them are drawn at random. If the six numbers drawn match the numbers that a player had chosen, the player wins \$1,000,000. If they match 5 numbers, then win \$1,000. It costs \$1 to buy a ticket. Find the expected value.

	outcome (x)	Probability $P(x)$	$x \cdot P(x)$
1st Place	$1{,}000{,}000 - 1 = \$999{,}999$	$\frac{_6C_6}{_{48}C_6} = \frac{1}{12{,}271{,}512}$	$\approx \$0.081\ldots$
2nd Place	$1{,}000 - 1 = \$999$	$\frac{_6C_5 \cdot {_{42}C_1}}{_{48}C_6} = \frac{252}{12{,}271{,}512}$	$\approx \$0.020\ldots$
Loss	$0 - 1 = -1$	$1 - \left(\frac{1}{12{,}271{,}512} + \frac{252}{12{,}271{,}512}\right)$	$\approx -\$0.999\ldots$

$$\frac{12{,}271{,}512 - (253)}{12{,}271{,}512} = \frac{12{,}271{,}259}{12{,}271{,}512}$$

* Note: we did these previous class

$$EV = \sum x\,P(x)$$

$$\approx (\$0.081) + (\$0.020) + (-\$.999)$$

$$\approx -\$0.898$$

$$\boxed{EV \approx -\$0.90}$$

→ expect Lose \$.90 per ticket on average

Try it Now 13

A friend offers to play a game, in which you roll 3 standard 6-sided dice. If all the dice roll different values, you give him $1. If any two dice match values, you get $2. What is the expected value of this game? Would you play?

	Outcomes x	Probability $P(x)$	$x\,P(x)$
win	2	$\frac{4}{9}$	$\frac{8}{9}$
loss	-1	$1-\frac{4}{9}=\frac{5}{9}$	$-\frac{5}{9}$

* by Counting Principle $\underline{6}\cdot\underline{6}\cdot\underline{6} = 216$ outcomes

111		
112	121	211
113	131	311
114	141	411
115	151	511
116	161	611

16 cases for 1 repeated

→ by similar logic, 16 cases for each of the 6 events

$P(\text{win})\ \frac{16\cdot 3}{216} = \frac{16}{36} = \frac{4}{9}$

$P(\text{Lose}) = 1-\frac{4}{9} = \frac{9}{9}-\frac{4}{9} = \frac{5}{9}$

$EV = \sum x\cdot P(x)$

$= \left(-\frac{5}{9}\right) + \left(\frac{8}{9}\right) = \frac{3}{9} = \frac{1}{3}$

$\approx +\$0.33$

* Expect to ~~gain~~ $.33 per play

Play on!

Example 44

A 40-year-old man in the U.S. has a 0.242% risk of dying during the next year[2]. An insurance company charges \$275 for a life-insurance policy that pays a \$100,000 death benefit. What is the expected value for the person buying the insurance?

	Outcome (x)	P(x)	x·P(x)
live	0 − \$275 = \$−275	1 − 0.00242 = .99758	≈ −\$274.3345
die	\$100,000 − \$275 = \$99,725	0.00242	≈ \$241.3345

$EV = \sum x \cdot P(x)$

$= (-\$274.3345) + (\$241.3345)$

$\boxed{EV = -\$33}$

Try it Now 13

A friend offers to play a game, in which you roll 3 standard 6-sided dice. If all the dice roll different values, you give him \$1. If any two dice match values, you get \$2. What is the expected value of this game? Would you play?

13. Suppose you roll the first die. The probability the second will be different is $\frac{5}{6}$. The probability that the third roll is different than the previous two is $\frac{4}{6}$, so the probability that the three dice are different is $\frac{5}{6}\cdot\frac{4}{6}=\frac{20}{36}$. The probability that two dice will match is the complement, $1-\frac{20}{36}=\frac{16}{36}$.

The expected value is: $(\$2)\cdot\frac{16}{36}+(-\$1)\cdot\frac{20}{36}=\frac{12}{36}\approx\$0.33$. Yes, it is in your advantage to play. On average, you'd win \$0.33 per play.