Example 34

A 4 digit PIN number is selected. What is the probability that there are no repeated digits?

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}$ ← no repetition

$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$ ← with repetition

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = {}_{10}P_4 = 5040$

$10^4 = 10{,}000$

$$P(\text{no repeats}) = \frac{{}_{10}P_4}{10^4} = \frac{5040}{10000} = 0.504$$

$$* \quad {}_nP_r = \frac{n!}{(n-r)!} = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

no replace

Example 35

In a certain state's lottery, 48 balls numbered 1 through 48 are placed in a machine and six of them are drawn at random. If the six numbers drawn match the numbers that a player had chosen, the player wins \$1,000,000. In this lottery, the order the numbers are drawn in doesn't matter. Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket.

number of possible outcomes:

* no repeat → 48^6 is not an option
* order doesn't matter → combination

$$_{48}C_6 = \frac{48!}{6!(48-6)!} = \frac{48!}{6!\,42!}$$

$$= \frac{48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43 \cdot \cancel{42!}}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{42!}}$$

$$= \frac{\cancel{48} \cdot 47 \cdot 46 \cdot \cancel{45}^{3} \cdot 44 \cdot 43}{\cancel{48} \cdot \cancel{15}}$$

$$= 47 \cdot 46 \cdot 3 \cdot 44 \cdot 43$$

$$= 12{,}271{,}512$$

winning combination: 6 numbers
6 chosen

$$_6C_6 = \frac{6!}{6!(6-6)!} = \frac{\cancel{6!}}{\cancel{6!}\,0!} = \frac{1}{0!} = \frac{1}{1} = 1$$

$$P(\text{winning}) = \frac{1}{12{,}271{,}512}$$

Example 36

In the state lottery from the previous example, if five of the six numbers drawn match the numbers that a player has chosen, the player wins a second prize of $1,000. Compute the probability that you win the second prize if you purchase a single lottery ticket.

number of possible outcomes: ${}_{48}C_6 = 12\ 271\ 512$

* from earlier.

how many combinations, choosing 5 correct out of 6

winning numbers overall: 6

chosen numbers: 5

${}_6C_5$

# of losing numbers $48-6=42$

one chosen : 1

${}_{42}C_1$

*Note $6+42 : 48$ overall # of balls

$5+1 : 6$ # chosen balls

By Counting Principle: $({}_6C_5)({}_{42}C_1)$

$${}_6C_5 = \frac{6!}{5!(6-5)!} = \frac{6!}{5!\,1!} = \frac{6\cdot\cancel{5!}}{\cancel{5!}\cdot 1} = 6$$

number of combinations in which 5 balls are correct of 6

$${}_{42}C_1 = \frac{42!}{1!(42-1)!} = \frac{42!}{1!\ 41!} = \frac{42\cdot\cancel{41!}}{1\cdot\cancel{41!}} = 42$$

number of combinations in which 1 balls are incorrect of 42 incorrect

$$({}_6C_5)({}_{42}C_1) = 6\cdot 42 = 252$$

$$P\left(\frac{5}{6}\right) = \frac{252}{12\ 271\ 512}$$

Example 37

Compute the probability of randomly drawing five cards from a deck and getting exactly one Ace.

overall: 52 cards
5 chosen $\rightarrow$ $_{52}C_5$
$= 2598960$

4 not A 1 A

$P = \dfrac{_{48}C_4 \cdot {_4C_1}}{_{52}C_5}$

5 cards chosen

Pick an A: 4 Aces
1 Ace $\rightarrow$ $_4C_1 = \dfrac{_4P_1}{1!}$

$= \dfrac{4}{1} = 4$

Not an A: $52 - 4 = 48$ not Ace
$5 - 1 = 4$ Chosen $\rightarrow$ $_{48}C_4 = \dfrac{_{48}P_4}{4!}$

$= \dfrac{48 \cdot 47 \cdot 46 \cdot 45}{4 \cdot 3 \cdot 2 \cdot 1}$

$= \dfrac{\overset{2}{\cancel{48}} \cdot 47 \cdot 46 \cdot 45}{\cancel{24}}$

$= 2 \cdot 47 \cdot 46 \cdot 45$

$= 194{,}580$

$P = \dfrac{_{48}C_4 \cdot {_4C_1}}{_{52}C_5} = \dfrac{194{,}580 \cdot 4}{2{,}598{,}960} = \dfrac{778320}{2{,}598{,}960} = \dfrac{3243}{10{,}829}$

Example 38

Compute the probability of randomly drawing five cards from a deck and getting exactly two Aces.

overall : 52 cards
5 chosen $\rightarrow$ $_{52}C_5$
$= 2598960$

3 not A 2A

$P = \dfrac{_{48}C_3 \cdot {_4C_2}}{_{52}C}$

5 cards chosen

Pick 2 A : 4 Aces
2 Ace $\rightarrow$ $_4C_2 = \dfrac{_4P_2}{2!}$

$= \dfrac{4 \cdot 3}{2 \cdot 1} = 6$

Not an A: $52 - 4 = 48$ not Ace
$5 - 2 = 3$ chosen $\rightarrow$ $_{48}C_3 = \dfrac{48 \cdot 47 \cdot 46}{3 \cdot 2 \cdot 1}$

$= \dfrac{\cancel{48}^{8} \cdot 47 \cdot 46}{\cancel{6}}$

$= 8 \cdot 47 \cdot 46$

$= 17\,296$

$P = \dfrac{_{48}C_3 \quad _4C_2}{_{52}C_5} = \dfrac{17296 \cdot 6}{2598960} = \dfrac{103776}{2598960}$

$= \boxed{\dfrac{2162}{54145}}$

Try it Now 10

Compute the probability of randomly drawing five cards from a deck of cards and getting three Aces and two Kings.

overall: 52 cards
5 chosen $\rightarrow$ $_{52}C_5$
$= 2598960$

Pick 3A $\rightarrow$ 4A
3 Chosen $\rightarrow$ $_4C_3 = \frac{4 \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3} \cdot \cancel{2} \cdot 1} = 4$

Pick 2K $\rightarrow$ 4K
2 chosen $\rightarrow$ $_4C_2 = \frac{4 \cdot 3}{2 \cdot 1} = 6$

* Note: Not remaining. just kings

$$P(3A, 2K) = \frac{_4C_3 \cdot {_4C_2}}{_{52}C_5} = \frac{4 \cdot 6}{2598960} = \frac{24}{2{,}598{,}960}$$

$$= \boxed{\frac{1}{108{,}290}}$$