Example 12

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

a) Has a red car *and* got a speeding ticket

b) Has a red car *or* got a speeding ticket.

	Speeding ticket	No speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

a.) $\dfrac{15 \text{ red and speeding}}{665 \text{ respondents}} = \dfrac{15}{665} = \dfrac{3}{133}$

b.)

	Speeding ticket	No speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

$$\frac{150}{665} + \frac{60}{665} - \frac{15}{665} = \frac{195}{665}$$

$$P(\text{Red}) + P(\text{ticket}) - P(\text{Red \& Ticket})$$

$$\frac{15 + 45 + 135}{665} = \frac{195}{665} \quad P(\text{Red or Ticket})$$

$$P(\text{Red or Ticket}) = 1 - \frac{470}{665} = \frac{665}{665} - \frac{470}{665}$$

$$= \frac{195}{665}$$

Conditional Probability

Often it is required to compute the probability of an event given that another event has occurred.

Example 13

What is the probability that two cards drawn at random from a deck of playing cards will both be aces?

→ Assumption: there is no replacement after 1st card is drawn

1st draw: $\frac{4 \text{ ACES}}{52 \text{ cards}}$

2nd draw: $\frac{3 \text{ Aces}}{51 \text{ cards}}$ remaining

assumes picked an ace on first draw.

Ace <u>and</u> Ace "$P(E_1) \cdot P(E_2)$"

$$\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

$$P(A) \cdot P(A \mid A \text{ on first try})$$

Probability of Ace on given Ace on first try

Conditional Probability

The probability the event B occurs, given that event A has happened, is represented as $P(B \mid A)$

This is read as "the probability of B given A"

Example 14

Find the probability that a die rolled shows a 6, given that a flipped coin shows a head.

$P(6) = \frac{1}{6}$ $P(H) = \frac{1}{2}$

Recall independent events: one event doesn't affect the other

not mutually exclusive: $P(A \text{ and } B) = 0$

$P(6|H) = \frac{1}{6}$ b/c there is no effect of coin flip on dice roll.

→ probability does not change.

Example 15

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

a) Has a speeding ticket *given* they have a red car

b) Has a red car *given* they have a speeding ticket

	Speeding ticket	No speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

a.) # red cars = 150

speeding ticket and red car: 15

$$P(\text{ticket} \mid \text{red}) = \frac{15}{150} = \frac{1}{10}$$

b.) # speeding tickets : 60

red & speeding : 15

$$P(\text{red} \mid \text{ticketed}) = \frac{15}{60} = \frac{1}{4}$$

$$* \; P(A \mid B) \neq P(B \mid A)$$

Conditional Probability Formula

If Events A and B are not independent, then

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Example 16

If you pull 2 cards out of a deck, what is the probability that both are spades?

no replacement after first draw

\#spades = 13

\#cards = 52

2nd draw assumes success in first draw

$$P(\spadesuit \text{ then } \spadesuit) = P(\spadesuit_1) \cdot P(\spadesuit \mid \spadesuit_1)$$

$$= \left(\frac{13\spadesuit}{52}\right)\left(\frac{12\spadesuit}{51}\right) = \frac{156}{2652} = \frac{1}{17}$$

$$\approx .0588$$