Example 6

Suppose we flipped a coin and rolled a die, and wanted to know the probability of getting a head on the coin and a 6 on the die.

Outcomes for coin: H, T

Outcomes for die: 1, 2, 3, 4, 5, 6

Sample space: (using table)

C \ D	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

S: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

one outcome

$|S| = 12$

# of outcomes

$$P(H6) = \frac{1 \text{ outcome}}{12 \text{ possible outcomes}} = \frac{1}{12}$$

$$P(H) = \frac{1}{2} \qquad P(6 \text{ on die}) = \frac{1}{6}$$

$$P(H \text{ and } 6) = P(H) \cdot P(6)$$

$$= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

Independent Events

Events A and B are **independent events** if the probability of Event B occurring is the same whether or not Event A occurs.

Example 7

Are these events independent?

a) A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head. Independent, b/c same coin, $P(H)$ is still same.

b) The two events (1) "It will rain tomorrow in Houston" and (2) "It will rain tomorrow in Galveston" (a city near Houston).
not independent. b/c of location, if raining in Houston just as likely for Galveston.

c) You draw a card from a deck, then draw a second card without replacing the first.

not independent b/c second deck has one fewer card.

***P*(*A* and *B*) for independent events**

If events A and B are independent, then the probability of both A and B occurring is

$P(A \text{ and } B) = P(A) \cdot P(B)$

where $P(A \text{ and } B)$ is the probability of events A and B both occurring, $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring

Example 8

In your drawer you have 10 pairs of socks, 6 pairs of which are white, and 7 tee shirts, 3 of which are white. If you randomly reach in and pull out a pair of socks and a tee shirt, what is the probability both are white?

$P(\text{white socks}) = \frac{6}{10} = \frac{3}{5}$ $P(\text{white t-shirt}) = \frac{3}{7}$

$P(\text{white socks } \underline{\text{and}} \text{ white t-shirt}) = P(\text{white socks}) \cdot P(WT)$

$= \frac{6}{10} \cdot \frac{3}{7}$

$= \frac{18 \div 2}{70 \div 2}$ $\frac{9}{35}$

independent event

Try it Now 2

A card is pulled a deck of cards and noted. The card is then replaced, the deck is shuffled, and a second card is removed and noted. What is the probability that both cards are Aces?

Aces = A♡, A♢, A♠, A♣

$\frac{4 \text{ Aces}}{52 \text{ cards}}$

w/replacement

$$P(A \text{ and } A) = P(A) \cdot P(A)$$

$$= \frac{4}{52} \cdot \frac{4}{52}$$

$$= \frac{16}{2704} \underset{\div 16}{\overset{\div 16}{=}} \frac{1}{169}$$

Example 9

Suppose we flipped a coin and rolled a die, and wanted to know the probability of getting a head on the coin *or* a 6 on the die.

C \ D	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

$\leftarrow P(H \text{ and } 6) = \frac{1}{12}$

$$P(\text{Head or } 6) = \frac{7}{12}$$

$P(H) = \frac{1}{2}$ $\qquad$ $P(6) = \frac{1}{6}$

$P(H \underline{\underline{or}} 6) = P(H) \underline{\underline{+}} P(6) - P(H \text{ and } 6)$

$= \frac{(3)}{(6)}\cancel{\frac{1}{2}} + \frac{1}{6} - \frac{1}{12}$

LCD = 6

$= \frac{3}{6} + \frac{1}{6} - \frac{1}{12}$

$= \frac{4}{6}\frac{2}{2} - \frac{1}{12}$

$= \frac{8}{12} - \frac{1}{12} = \frac{7}{12}$

***P*(*A* or *B*)**

The probability of either *A* or *B* occurring (or both) is

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 10

Suppose we draw one card from a standard deck. What is the probability that we get a Queen or a King?

$P(K) = \frac{4}{52}$

$P(Q) = \frac{4}{52}$

$P(Q \text{ and } K) = \frac{0}{52} = 0$

$P(Q \text{ or } K) = P(Q) + P(K) - P(Q \text{ and } K)$

$= \frac{4}{52} + \frac{4}{52} - 0$

$= \frac{8}{52} \begin{matrix}\div 4 \\ = \\ \div 4\end{matrix} \frac{2}{13}$

In the last example, the events were **mutually exclusive**, so $P(A \text{ or } B) = P(A) + P(B)$.

mutually exclusive events – events that cannot happen at the same time.

e.g. light on and off

win / lose

9 and J one card

Example 11

Suppose we draw one card from a standard deck. What is the probability that we get a red card or a King?

$P(\text{Red}) = \frac{26}{52} \cdot \frac{1}{2}$

$P(K) = \frac{4}{52} = \frac{1}{13}$ 2R, 2B

$P(\text{Red } K) = \frac{2}{52}$

$P(\text{Red or King}) = P(\text{Red}) + P(K) - P(\text{Red } K)$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$

$= \frac{28}{52} \overset{\div 4}{\underset{\div 4}{=}} \frac{7}{13}$

pairs

Try it Now 3

In your drawer you have 10 pairs of socks, 6 of which are white, and 7 tee shirts, 3 of which are white. If you reach in and randomly grab a pair of socks and a tee shirt, what the probability at least one is white?

$$P(WS) = \frac{6}{10}$$

$$P(WT) = \frac{3}{7}$$

$$P(WS \text{ and } WT) = P(WS) \cdot P(WT)$$

$$= \frac{6}{10} \cdot \frac{3}{7}$$

$$= \frac{18}{70}$$

$$P(WS \text{ OR } WT) = P(WS) + P(WT) - P(WS \text{ AND } WT)$$

$$= \frac{6}{10} + \frac{3}{7} - \frac{18}{70}$$

$$= \frac{42}{70} + \frac{30}{70} - \frac{18}{70}$$

$$= \frac{54}{70}$$

$P(\text{not WS}) = \frac{4}{10}$ $\qquad$ $P(\text{not WT}) = \frac{4}{7}$

$$P(\text{WS or WS}) = 1 - P(\text{not WS AND not WT})$$

$$= 1 - P(\text{not WS}) \cdot P(\text{not WT})$$

$$= 1 - \frac{4}{10} \cdot \frac{4}{7}$$

$$= \frac{70}{70}\cancel{1} - \frac{16}{70}$$

$$= \frac{54}{70}$$