Events and Outcomes

The result of an experiment is called an **outcome**.

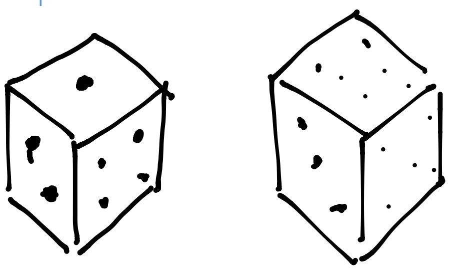
An **event** is any particular outcome or group of outcomes.

A **simple event** is an event that cannot be broken down further

The **sample space** is the set of all possible simple events.

Example 1

If we roll a standard 6-sided die, describe the sample space and some simple events.



Outcomes: {1, 2, 3, 4, 5, 6}

sample space

event: even numbers
odd numbers

sample space {even, odd}

event: less than 2 : 1
greater than 4 : 5, 6

less than 7: 1, 2, 3, 4, 5, 6
greater than 7: no outcomes
but that still is
an event.

Basic Probability
Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$P(E) = \frac{\text{Number of outcomes corresponding to the event E}}{\text{Total number of equally - likely outcomes}}$$

Probability of # greater than 4.

$$P(x>4) = \frac{2}{6} = \frac{1}{3}$$

Event: has 2 outcomes: 5, 6

overall # of outcomes: 6 $\{1,2,3,4,5,6\}$

Example 2

If we roll a 6-sided die, calculate
a) P(rolling a 1)
b) P(rolling a number bigger than 4)

a.) $P(x=1) = \frac{1}{6}$ ← one outcome that produces "1"; ← overall number of outcomes

b.) We just discussed.

the rest are sour.

Example 3

Let's say you have a bag with 20 cherries, 14 sweet ~~and 6 sour~~. If you pick a cherry at random, what is the probability that it will be sweet?

$$P(\text{sweet}) = \frac{14 \text{ sweet}}{20 \text{ cherries}} = \frac{7}{10}$$

$$P(\text{sour}) = \frac{6 \text{ sour}}{20 \text{ cherries}} = \frac{3}{10}$$

$$= \frac{20 \text{ cherries} - 14 \text{ sweet}}{20 \text{ cherries}} = \frac{3}{10}$$

$$= 1 - P(\text{sweet}) = 1 - \frac{7}{10} = \frac{10}{10} - \frac{7}{10} = \frac{3}{10}$$

Cards

A standard deck of 52 playing cards consists of four **suits** (hearts, spades, diamonds and clubs). Spades and clubs are black while hearts and diamonds are red. Each suit contains 13 cards, each of a different **rank**: an Ace (which in many games functions as both a low card and a high card), cards numbered 2 through 10, a Jack, a Queen and a King.

Example 4

Compute the probability of randomly drawing one card from a deck and getting an Ace.

A♡ A♢

A♣ A♠

$$P(A) = \frac{4 \text{ Aces}}{52 \text{ cards}}$$

4 outcomes of an Ace out of 52 cards

$$P(\text{not } A) = \frac{48}{52} = \frac{\overset{\text{total}}{52} - \overset{A}{4}}{52}$$

$$= \frac{52}{52} - \frac{4}{52} = 1 - P(A)$$

P(Draw a Card): $\frac{52 \text{ outcomes}}{52 \text{ cards}}$: 1 100%

Max Probability: 1 ← certainty

Min Probability: 0 ← impossibility

Certain and Impossible events
An impossible event has a probability of 0.
A certain event has a probability of 1.
The probability of any event must be $0 \le P(E) \le 1$

Complementary Probability

$P(\text{not } A) = \underbrace{1}_{\text{certainty}} - P(A)$

Complement of an Event
The **complement** of an event is the event "E doesn't happen"
The notation $\bar{E}$ is used for the complement of event E.
We can compute the probability of the complement using $P(\bar{E}) = 1 - P(E)$
Notice also that $P(E) = 1 - P(\bar{E})$

Example 5

If you pull a random card from a deck of playing cards, what is the probability it is not a heart?

There are 13 hearts in the deck, so $P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$.

The probability of *not* drawing a heart is the complement:

$P(\text{not heart}) = 1 - P(\text{heart}) = 1 - \frac{1}{4} = \frac{3}{4}$