Example 1 Comparing Simple and Compound Interest

Suppose that $5,000 is invested for 3 years at 8%.

(a) Find the amount of simple interest.

(b) Find the compound interest if interest is calculated once per year.

a.) $I = ?$

$P = 5000$

$r = 8\% = .08$

$t = 3$

$I = Prt$

$I = (5000)(.08)(3)$

$I = \$1200$

b.) $A = P + I$

$A = P + Prt$

$A = P(1 + rt)$

$I = ?$

$P_0 = 5000$

$r = 8\% = .08$

$t = 1$

end in 3 years

$A = 5000(1 + (.08)(1))$

$A = 5000(1.08) = \$5400 = P_1$

Final Value after 1 year

$P_2 = P_1 (1.08)$

$= (\$5400)(1.08) = \$5832$

$P_3 = P_2 (1.08)$

$= (\$5832)(1.08) = \underline{\$6298.56}$

↑ Final Value after 3 years

$A = P + I$

$\$6298.56 - \$5000$

$A - P_0 = I$

$P_N - P_0 = I$ →

↓

$I = \$1298.56$

compound interest

$P_0 = 5000 = 5000(1.08)^0$

$P_1 = 5400 = 5000(1.08) = 5000(1.08)^1$

$P_2 = 5832 = 5400(1.08) = 5000(1.08)(1.08) = 5000(1.08)^2$

$P_3 = 6298.56 = 5832(1.08) = 5000(1.08)(1.08)(1.08) = 5000(1.08)^3$

$\vdots$

$P_5 = 5000(1.08)^5$

$P_N = 5000(1.08)^N \longrightarrow$ **$P_N = P_0(1+r)^N$**

N: # terms years.

r: compounding once over the course of a year

Example 4

A certificate of deposit (CD) is a savings instrument that many banks offer. It usually gives a higher interest rate, but you cannot access your investment for a specified length of time. Suppose you deposit $3000 in a CD paying 6% interest, compounded monthly. How much will you have in the account after 20 years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $\qquad$ $P_N = P_0\left(1+\frac{r}{k}\right)^{Nk}$

* Never tells you what the interest is.

Use $A = P+I$ to find interest

$A = P_N$ = Final Value $= ?$

$P = P_0$ = Initial Value $= \$3000$

r = annual rate $= .06$

$n = k$ = number of times compounded $= 12$

$t = N$ = number of years $= 20$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= (\$3000)\left(1+\frac{(.06)}{(12)}\right)^{(12)(20)}$$

$$= \$3000\left(1+.005\right)^{240}$$

$$= \$3000\left(1.005\right)^{240}$$

$$= \$3000\,(3.310204475\,81) \qquad \sim 3000(3.3102)$$

$$\approx \$9930.613 \approx \$9930.61$$

$I = \$9{,}930.61 - \$3{,}000 = \$6930.61$

Example 4 **Computing Compound Interest**

Find the interest on $11,000 compounded daily at 5% for 6 years. Assume a 365-day year.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 11000$

$r = 0.05$

$n = 365$

$t = 6$

$A = (11000)\left(1 + \frac{(0.05)}{(365)}\right)^{(365)(6)}$

$A \approx 11000\,(1.000136986)^{2190}$

$A \approx (11000)(1.349831074)$

$A \approx \underline{\$14{,}848.14}$

total value

$A - P = I$

$\$14{,}848.14 - \$11{,}000 = \underline{\$3{,}848.14}$

interest

Example 5

You know that you will need $40,000 for your child's education in 18 years. If your account earns 4% compounded quarterly, how much would you need to deposit now to reach your goal?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = \$40{,}000$

$P = ?$

$r = 4\% = 0.04$

$n = 4$

$t = 18$ years

$(\$40000) = P\left(1+\frac{(0.04)}{(4)}\right)^{(4)(18)}$

$40000 = P\left(1+\frac{.04}{4}\right)^{72}$

$40{,}000 = P(1+0.01)^{72}$

$40{,}000 = P(1.01)^{72}$

$40{,}000 \approx P\,(2.047099\,3121)$

$\frac{40{,}000}{2.047099\,3121} \approx \frac{P\,(2.047099\,3121)}{2.047099\,3121}$

$\$19{,}539.843 \approx P$

$\$19{,}539.84 \approx P$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$\left(\$40000\right) = P\left(1+\frac{(0.04)}{(4)}\right)^{(4)(18)}$

$\frac{A}{(1+\frac{r}{n})^{nt}} = P$

$40000 = P\left(1+\frac{.04}{4}\right)^{72}$

$40{,}000 = P\left(1+0.01\right)^{72}$

$\frac{40{,}000}{(1.01)^{72}} = \frac{P\cdot(1.01)^{72}}{(1.01)^{72}}$

$\$19{,}539.84 \approx P$

9. How much would you need to deposit in an account now in order to have $6,000 in the account in 8 years? Assume the account earns 6% interest compounded monthly.

10. How much would you need to deposit in an account now in order to have $20,000 in the account in 4 years? Assume the account earns 5% interest. compounded weekly.

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$6000 = P\left(1+\frac{.06}{12}\right)^{(12)(8)}$

$6000 = P\left(1.005\right)^{96}$

$6000 = P(1.614142708)$

$\frac{6000}{1.614142708} = P$

$P \approx \$3{,}717.14$

$A = 6000$

$r = .06$

$n = 12$

$t = 8$

$P = ?$

9. How much would you need to deposit in an account now in order to have $6,000 in the account in 8 years? Assume the account earns 6% interest compounded monthly.

10. How much would you need to deposit in an account now in order to have $20,000 in the account in 4 years? Assume the account earns 5% interest. weekly

$P = ?$

$A = \$20{,}000$

$r = .05$

$t = 4$

$n = 52$

$$\$20{,}000 = P\left(1 + \frac{.05}{52}\right)^{208}$$

$$20{,}000 = P\left(1.00009615\right)^{208}$$

$$20{,}000 \approx P(1.22128)$$

$$\frac{20{,}000}{1.22128} \approx P$$

$$\$16376.188 \approx P$$

$$\$16{,}376.19 \approx P$$